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A COMPUTER PROGRAM
TO DETERMINE GEOMETRIC
PARAMETERS FOR THE
A. E. M. SOLAR ARRAYS

EDWARD M. GADDY

APRIL 1974



— GODDARD SPACE FLIGHT CENTER —
GREENBELT, MARYLAND

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ABSTRACT

A computer program has been written to aid in the design of the A. E. M.-1 solar array and to determine the power that will finally be available from the array. The program will plot the array output as a function of the satellite's position in a given orbit and will calculate the average output over the orbit.

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CONTENTS

	<u>Page</u>
INTRODUCTION	1
DESCRIPTION OF THE A.E.M. AND ITS ORBIT.	1
PROGRAM OUTLINE AND STRATEGY.	2
ORIENTATION OF UNIT VECTORS	2
LINE BY LINE EXPLANATION OF THE PROGRAM	5
RESULTS OF THE PROGRAM AS APPLIED TO THE A.E.M.-1 . . .	6
ACKNOWLEDGEMENT	8
REFERENCE.	8
APPENDIX A	A-1
APPENDIX B	B-1
APPENDIX C	C-1
APPENDIX D	D-1
APPENDIX E	E-1
APPENDIX F	F-1
APPENDIX G	G-1
APPENDIX H	H-1

ILLUSTRATIONS

<u>Figure</u>		<u>Page</u>
1	The A.E.M. Satellite	2
2	Euler's Angles	4

A COMPUTER PROGRAM TO DETERMINE GEOMETRIC PARAMETERS FOR THE A.E.M. SOLAR ARRAYS

INTRODUCTION

A computer program has been written to aid in the design of the A.E.M.-1 solar array and to determine the power that will finally be available from the array. The program will plot the array output as a function of the satellite's position in a given orbit, and will calculate the average output over the orbit. To do this, the program needs to have only the satellite's paddle pitch angles, rotational position and orbit.

DESCRIPTION OF THE A.E.M. AND ITS ORBITS

In order to understand the program it is necessary to have a definition of the satellite and its orbit. This section will define these.

Figure 1 depicts the satellite and an attached coordinate system. The satellite always orbits such that the $-z$ axis points toward the center of the earth. The satellite's rotational position, $R\theta$, is defined as the angle between the satellite's velocity vector and the axis of paddle 1.

As shown in Figure 1, all the paddle axes are perpendicular to the z axis. Although it is not shown in the figure, any paddle may be pitched to any angle about its axis. Zero pitch is defined by the pitch of the paddles in Figure 1. A positive rotation in pitch is represented by an axial vector pointed away from the center of the spacecraft.

In general, the A.E.M. satellite can have any orbit, and thus the program must be able to concern itself with any possibility. However, the A.E.M.-1 orbit is the specific case under consideration here; therefore, the program will be applied to this case. The generalization to all cases will be obvious.

The A.E.M.-1 has the following orbital parameters.

Inclination: 97° , ascending node

Hour: 2:00 p. m.

β = Beta: (The angle between the satellite's orbital angular momentum and a line from the earth to the sun) 55° to 67°

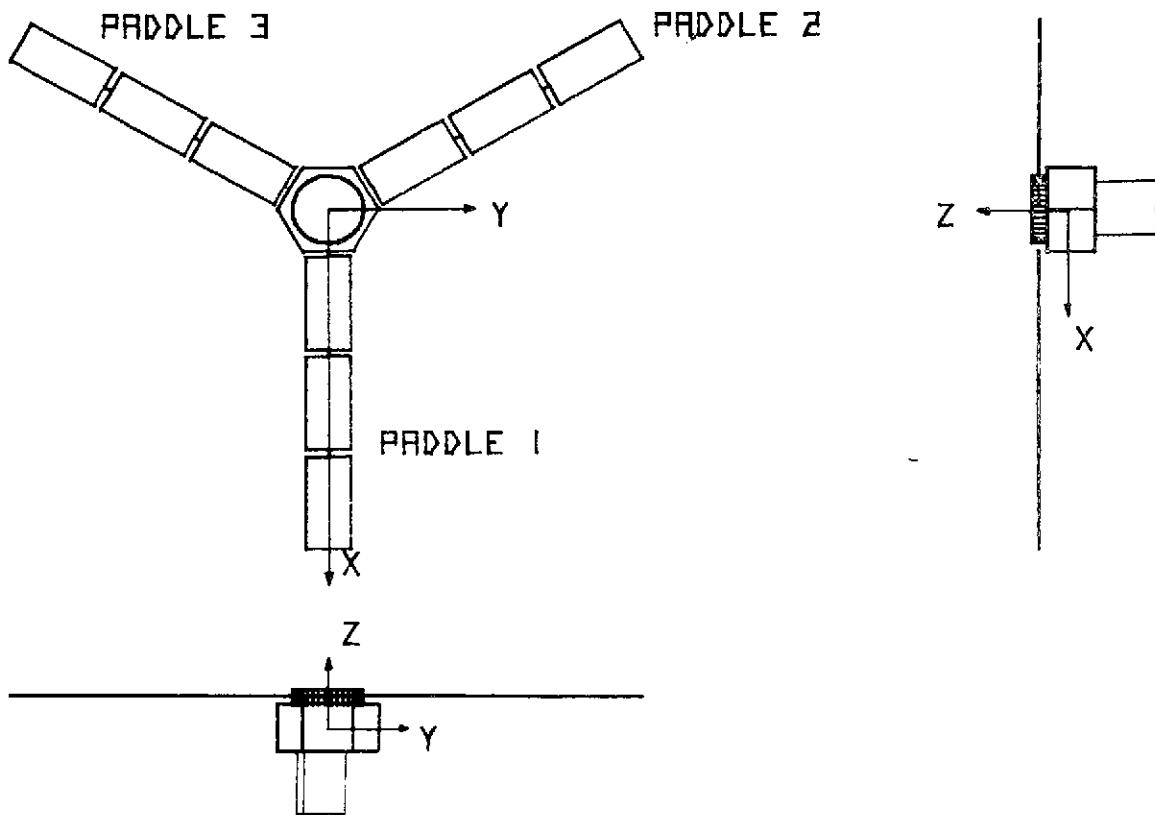


Figure 1. The A.E.M. Satellite

As far as the solar array is concerned, the only relevant parameter is β . The inclination and hour of the orbit do not explicitly affect the solar array output. As a result, it is convenient to arbitrarily choose all orbits to have an inclination of 90° , but with the same β as the actual orbit. This results in no change in the solar array output, but yields an easier programming problem. (The following consideration shows that this is true. Any orbit can be made polar by mathematically rotating the earth's axis in the plane of the terminator so that if extended it would intersect the satellite's orbit. By assumption this involves no change in the array output or β , and gives an orbit with a 90° inclination.)

PROGRAM OUTLINE AND STRATEGY

What the program does is take a given β angle, a given set of paddle pitch angles, a given satellite rotational position, and plot the projected area the solar array presents to the sun over the given orbit (as defined by β), simultaneously computing the average for the orbit. The projected area is given in terms of the area of a single paddle. This projected area may be multiplied by the output of

a single paddle in order to compute the array power output. The final plots so generated by the program are shown in Appendix F.

The program strategy is to orient unit vectors normal to each paddle for any specified set of conditions, and then to take the absolute value of the projection of each vector on the sunline and finally to sum these scalars. (This sum is the projected area of the array for the specified conditions.) Thus the most difficult problem faced by the program is how to orient the unit vectors properly.

ORIENTATION OF UNIT VECTORS

To describe how the unit vectors are oriented, I shall first describe the scheme the program uses to rotate vectors and then give the sequence in which the program performs rotations.

The program uses Euler's angles to perform all rotations. The specific set used is defined in Figure 2.

The rotation operator \tilde{A} obtained from these angles is:

$$A^{-1} = \tilde{A} = \begin{pmatrix} \cos\psi \cos\phi & -\cos\theta \sin\phi \sin\psi & -\sin\psi \cos\phi & -\cos\theta \sin\phi \cos\psi & \sin\theta \sin\phi \\ \cos\psi \sin\phi & +\cos\theta \cos\phi \sin\psi & -\sin\psi \sin\phi & +\cos\theta \cos\phi \cos\psi & -\sin\theta \cos\phi \\ \sin\theta \sin\psi & & \sin\theta \cos\psi & & \cos\theta \end{pmatrix}$$

To be more specific, if one has a three dimensional vector defined by

$$S = \begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix},$$

then this vector may be rotated first through ϕ , then θ , then ψ to obtain a new vector S' . S' is given by

$$S' = \tilde{A} S.$$

The specific sequence followed by the program is as follows. A unit vector is defined along the z axis, shown in Figure 1. This vector is then rotated by the angle defining the rotational position of the satellite, $R\theta$, and is pitched an amount, P_1 , equal to the pitch of paddle 1. The resulting vector is stored. This stored vector corresponds to a unit vector normal to paddle 1 when the

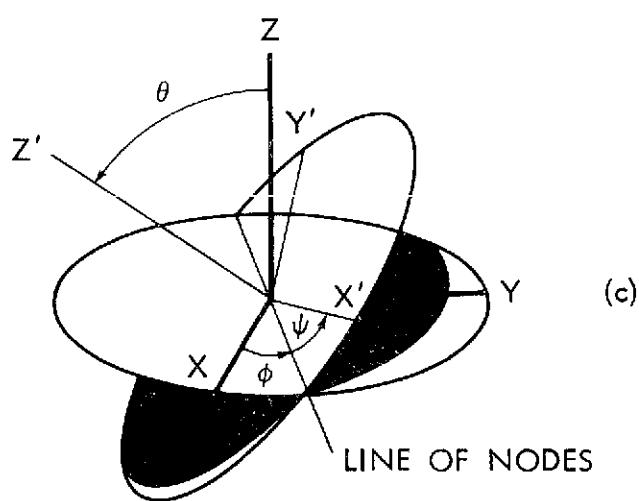
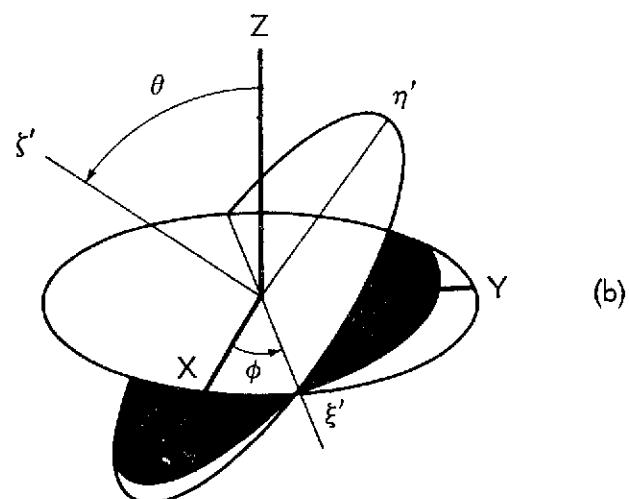
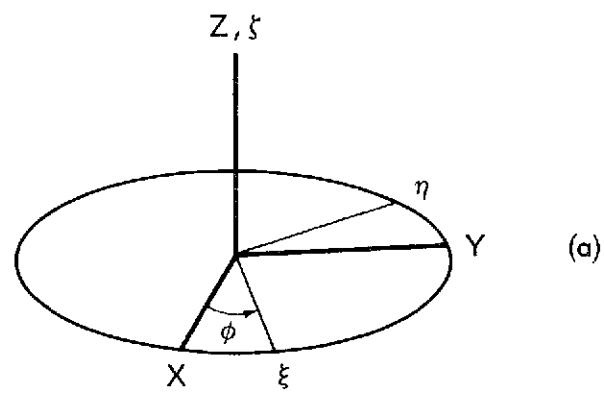


Figure 2. Euler's Angles

satellite is rotated by $R\theta$. The same procedure is followed for paddles 2 and 3 except that they are rotated by $(R\theta + 120^\circ)$ and $(R\theta + 240^\circ)$, respectively and pitched P2 and P3 respectively. It should be obvious from Figure 2 that the sequence of rotations described above can be accomplished for any one paddle by the matrix \tilde{A} where $R\theta$ corresponds to ϕ , where the paddle pitch angle corresponds to Θ , and where $\psi = 0$.

The three resulting vectors are then rotated -90° about the x axis, by a redefined \tilde{A} where $\phi = 0^\circ$, $\theta = -90^\circ$ and $\psi = 0^\circ$. Finally the three vectors are rotated by yet another \tilde{A} , where $\phi = 90^\circ - \beta$, $\theta = 90^\circ$ and ψ represents the satellite's orbit position. $\psi = 0^\circ$ obviously corresponds to the satellite being over the North Pole; $\psi = -90^\circ$ to the equator, and $\psi = -180^\circ$ to the South Pole.

LINE BY LINE EXPLANATION OF THE PROGRAM

In this section an explanation is given of the program, which is listed in Appendix A. The explanation will be done on a line by line basis. (The program is not structured so that a flow chart is necessary.) However, before this is done it should be pointed out that the program is in basic and will run on a Hewlett-Packard 9830A with a matrix operations R.O.M., a plotter R.O.M., a 9862A plotter, and a 9866 printer.

Lines 5-10 θ Input data, scale the plotter, and set the computer for degrees.

Line 11 θ Computes the number of degrees the satellite is in sunlight. We are given information, Appendix B, from the dynamics people on β (time) and shadow duration as a function of time. Thus we can obtain shadow duration as a function of β . From this, and the orbit period line 11 θ calculates the number of degrees out of 360° the satellite will be in sunlight or the range.

Lines 12 θ -13 θ Dimension the matrices used in the program (including some spares).

Line 14 θ Sets $A = \emptyset$, where A will be used as an accumulator in computing the average array output over an orbit.

Lines 15 θ -16 θ Define a unit vector in the +z direction.

Lines 17 θ -20 θ Take the original unit vector, rotate it $R\theta$, and pitch it P1. The subroutine 86 θ -95 θ define the matrix \tilde{A} derived from Euler's angles.

Lines 21 θ -24 θ Take the original vector, rotate it $R\theta + 120^\circ$ and pitch it P2.

Lines 250-280 Take the original vector, rotate it $R\phi +240^\circ$ and pitch it P3.

Lines 290-370 Rotate the three vectors (resulting from lines 170-280) -90° about the x axis. For this rotation, the matrix $\tilde{\Lambda}$, defined earlier in the report operating on a vector always results in the x component of the vector remaining unchanged, the original y component going to the new z component, and the original z component going to the new -y component.

Lines 380-400 Partially define the matrix operators for the final rotations. P is the computer's symbol for $\psi = 90^\circ - \beta$. The three matrix elements are defined here instead of in lines 420-470 because this keeps them out of the FOR . . . NEXT loop and thus saves computation time.

Line 410 Initiates a FOR . . . NEXT loop in which the satellite is rotated through its orbit. The loop starts with the satellite at a $+\psi$ value and rotates it over the North Pole $\psi = 0$, over the equator, $\psi = -90^\circ$ and over the South Pole, $\psi = -180^\circ$ to the point where the satellite enters darkness.

Lines 420-470 Completes the definition of the matrix operator.

Lines 480-500 Rotate the three vectors that were defined earlier by $\phi = 90^\circ - \beta$, $\theta = 90^\circ$, and ψ = satellite's position in orbit, as defined by the FOR . . . NEXT loop.

Line 510 Computes the absolute value of the projection of each vector onto the sunline and sums the result. The values C1, C2, and C3 are included in order to study one paddle at a time, i.e., by setting C1 = C2 = 0 only the projected area of paddle 3 is computed.

Line 520 Plots the results.

Line 530 Accumulates the results. After the orbit has been completed A/101 equals the average array output as per line 410.

Line 540 Completes the FOR . . . NEXT loop.

Lines 545-770 Label the resulting graph.

RESULTS OF THE PROGRAM AS APPLIED TO THE A.E.M.-1

We have used the program to find the optimum pitch angles for the A.E.M.-1 solar paddles as follows.

An orbit of $\phi = 30^\circ$ was chosen as representative of all possible A.E.M.-1 orbits, $\phi = 23^\circ$ to 35° , and the "C parameters" were chosen as:

$$C1 = 1$$

$$C2 = C3 = \phi.$$

This insured that only the output of paddle one would appear in the final graph. Paddle one was then pitched from 0° to 180° in 10° increments with a graph of the solar array projected area generated for each increment. The results of this study, shown in Appendix C show that a pitch of 90° on paddle one gives the highest possible output for that paddle in a $\phi = 30^\circ$ orbit.

Similar runs were made for paddles two and three, except that constraints from the experimenter limited the paddle pitch angle ranges to 70° thru 110° for paddle two, and 45° thru 135° for paddle three. The results of these studies, in Appendices D and E, show that paddle two has its optimum pitch angle at $+70^\circ$, and paddle three has its optimum pitch angle at $+135^\circ$.

Next, graphs were generated for $\phi = 23^\circ$, 30° , and 35° for the optimized paddle pitch angles. These graphs, in Appendix F, give the power output of the array exclusive of shadowing considerations.

The shadowing considerations are taken care of by another computer program which draws pictures of the satellite in any orbit position. These pictures are then analyzed for the amount of shadowing on each paddle. For completeness, this computer program has been included in Appendix G. The pictures generated by the program for $\phi = 35^\circ$ are in Appendix H. A report soon to be published will describe this program.

As can be seen for $\phi = 35^\circ$, shadowing considerations reduce the solar array output by 6.1 percent. Applying this same percentage to $\phi = 23^\circ$ and $\phi = 30^\circ$ yields average projected areas for each orbit of

$$1.63 \text{ for } \phi = 23^\circ$$

$$1.66 \text{ for } \phi = 30^\circ$$

$$1.68 \text{ for } \phi = 35^\circ$$

Assuming each solar paddle produces 77 W, this results in a minimum array output of 126 W. If high efficiency cells such as the "violet" cell are used instead of conventional cells the output is 151 W.

ACKNOWLEDGEMENT

I wish to acknowledge James A. Bass for his helpful discussions on the computer program and related geometry problems.

REFERENCE

1. Goldstein, Herbert. Classical Mechanics. Addison-Wesley Publishing Co., 1950.

APPENDIX A

22<

```

5 DISP "WHAT ARE AREA FACTORS C1,C2,C3";
6 INPUT C1,C2,C3
10 DISP "WHAT IS ROTATION";
20 INPUT R0
30 DEG
40 SCALE +160,-340,-0.4,3.2
50 DISP "WHAT IS P(1),P(2),P(3),PHI";
60 INPUT P1,P2,P3,PHI
70 DISP "WHAT IS THE DAY";
80 INPUT Z1
90 DISP "WHAT IS SHADOW TIME";
100 INPUT T
110 R=(97.85-T)*360/97.85
120 DIM A[3,3],B[3],C[3],D[3,3],E[3],F[3],G[3],H[3,3],J[3,3]
130 DIM K[3],L[3],M[3],N[3],O[3],P[3]
140 A=0
150 B[1]=B[2]=0
160 B[3]=1
170 A1=R0
180 A2=P1
190 GOSUB 860
200 MAT C=A*B
210 A1=R0+120
220 A2=P2
230 GOSUB 860
240 MAT E=A*B
250 A1=R0+240
260 A2=P3
270 GOSUB 860
280 MAT F=A*B
290 K[1]=C[1]
300 K[2]=C[3]
310 K[3]=-C[2]
320 L[1]=E[1]
330 L[2]=E[3]
340 L[3]=-E[2]
350 M[1]=F[1]
360 M[2]=F[3]
370 M[3]=-F[2]
380 D[3,3]=0
390 D[1,3]=SIMP
400 D[2,3]=-COSP
410 FOR S1=-(90-R/2) TO -(90+R/2) STEP -R/100
420 D[3,1]=SIN S1
430 D[3,2]=COS S1
440 D[1,1]=-D[3,2]*D[2,3]
450 D[1,2]=D[3,1]*D[2,3]
460 D[2,1]=D[3,2]*D[1,3]
470 D[2,2]=-D[3,1]*D[1,3]
480 MAT N=D*K
490 MAT O=D*L
500 MAT P=D*M

```

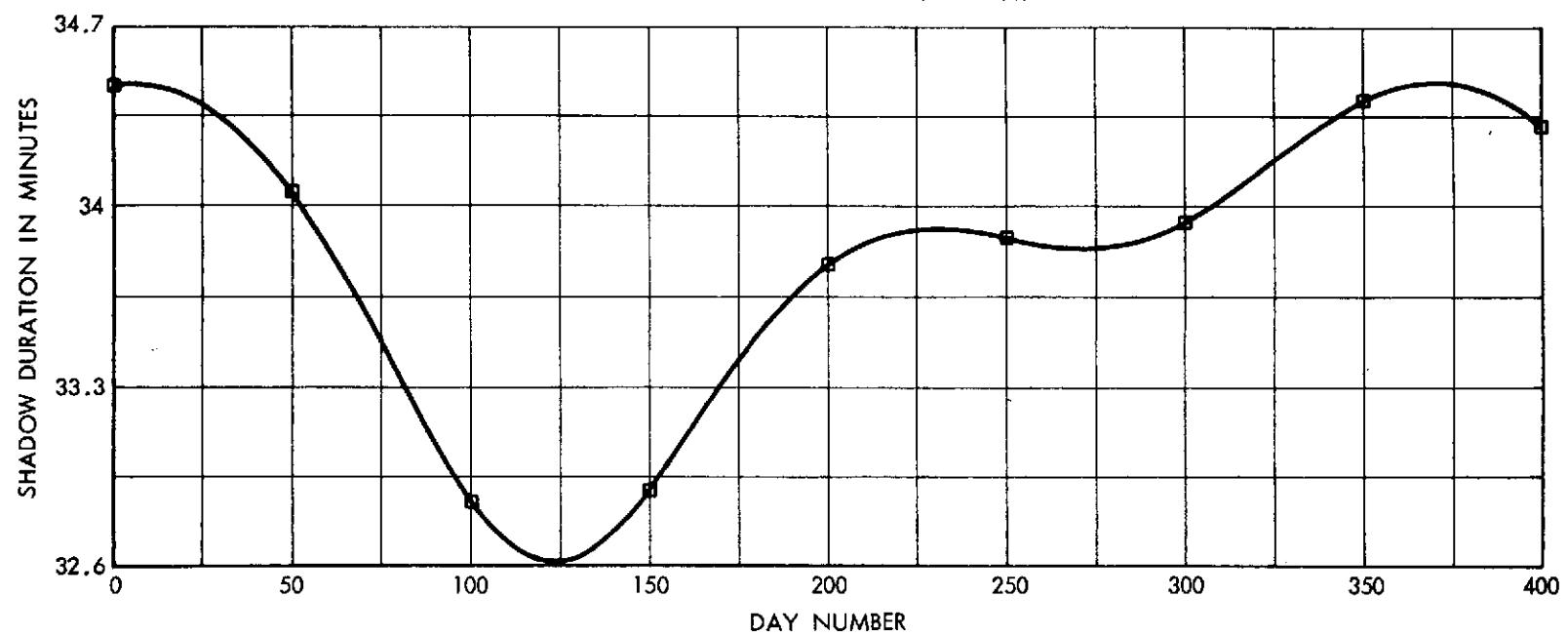
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510 E=C1*ABS(N[1])+C2*ABS(O[1])+C3*ABS(P[1])
520 PLOT S1,E,-2
530 R=R+E
540 NEXT S1
545 SCALE -160,340,-0.4,3.2
550 XAXIS 0,20,-100,+270
560 YAXIS -100,0.5,0.3
570 PLOT -20,3,1
580 LABEL (*,1.5,1.7,0,18/25)"A.E.M. PARAMETRIC ARRAY STUDY APRIL 21, 1974"
590 LABEL (*)"GADDY C1=";C1;" C2=";C2;" C3=";C3
600 LABEL (*)"P(1)=";P1;" P(2)=";P2;" P(3)=";P3;" PHI=";P
610 LABEL (620)"AVG.=";R/101,"RANGE=";R,"SHADOW TIME=";T
615 LABEL (*)"ROTATION=";R0
620 FORMAT F6.3,2X,F5.0,2X,F6.2
630 FOR I=80 TO -240 STEP -40
640 PLOT -I,0,1
650 CPLOT -2,-1.5
660 LABEL (*)I
670 NEXT I
680 PLOT 40,-0.3,1
690 LABEL (*)"PSI IN DEGREES"
700 FOR I=0 TO 3
710 PLOT -100,I,1
720 CPLOT -3,0
730 LABEL (*)I
740 NEXT I
750 PLOT -120,0.5,1
760 LABEL (*,1.5,1.7,90,18/25)"NORMALIZED PADDLE AREA"
770 PLOT 330,3.2,1
780 END
860 R[1,1]=COSA1
870 R[1,2]=-COSA2*SINA1
880 R[1,3]=SINA2*SINA1
890 R[2,1]=SINA1
900 R[2,2]=COSA2*COSA1
910 R[2,3]=-SINA2*COSA1
920 R[3,1]=0
930 R[3,2]=SINA2
940 R[3,3]=COSA2
950 RETURN

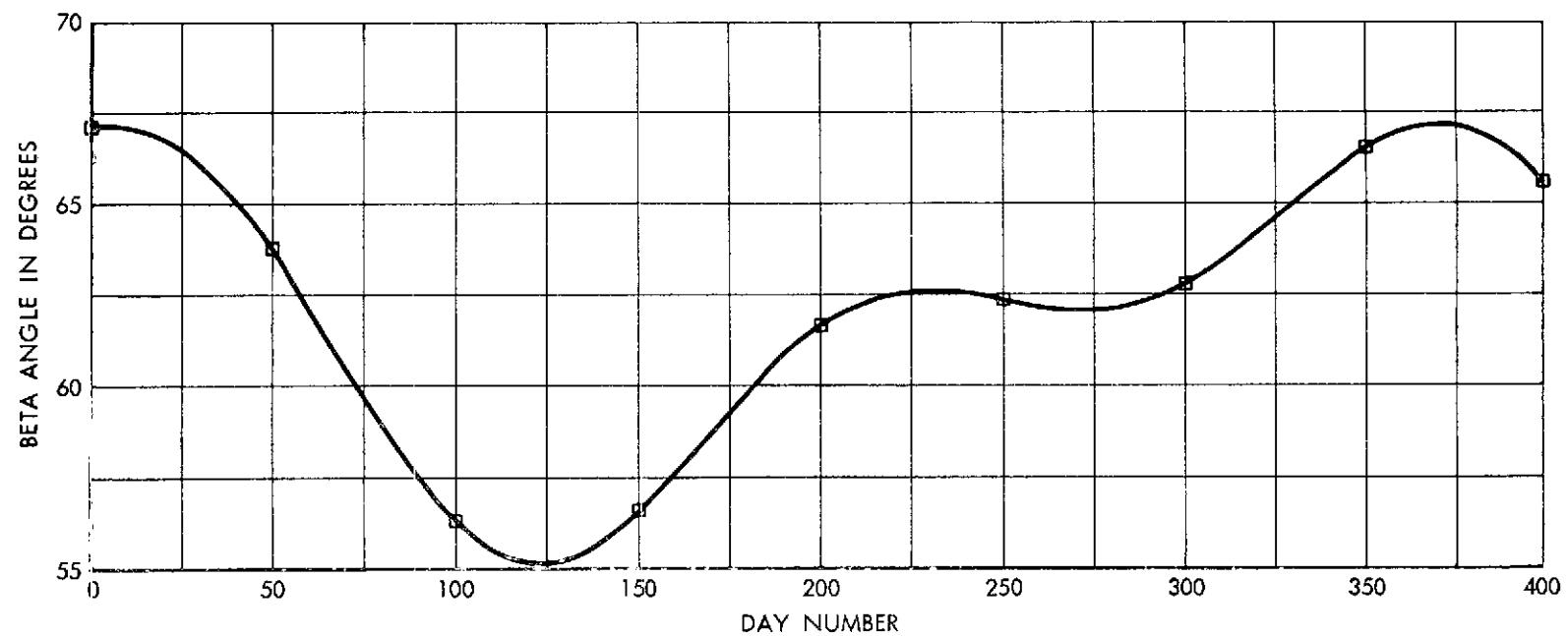
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APPENDIX B

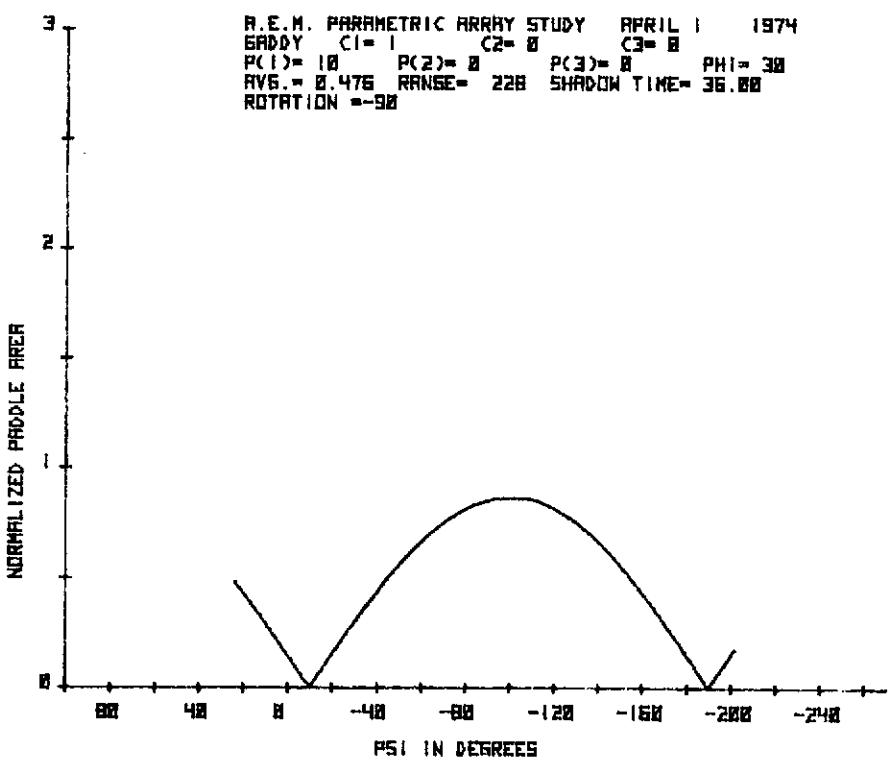
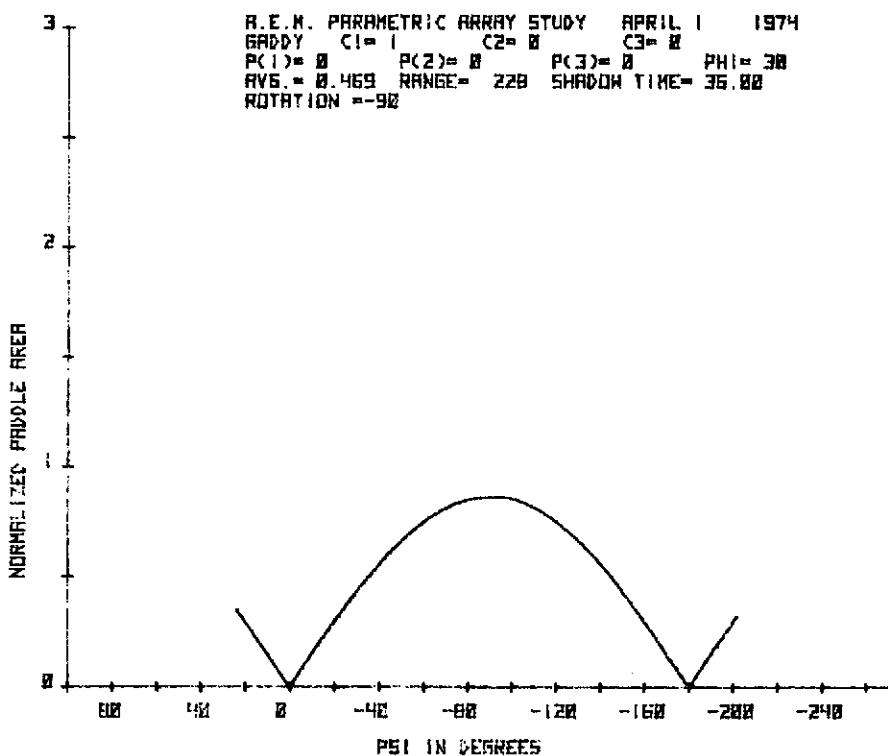
SHADOW DURATION VS DAY NUMBER A.E.M.-1



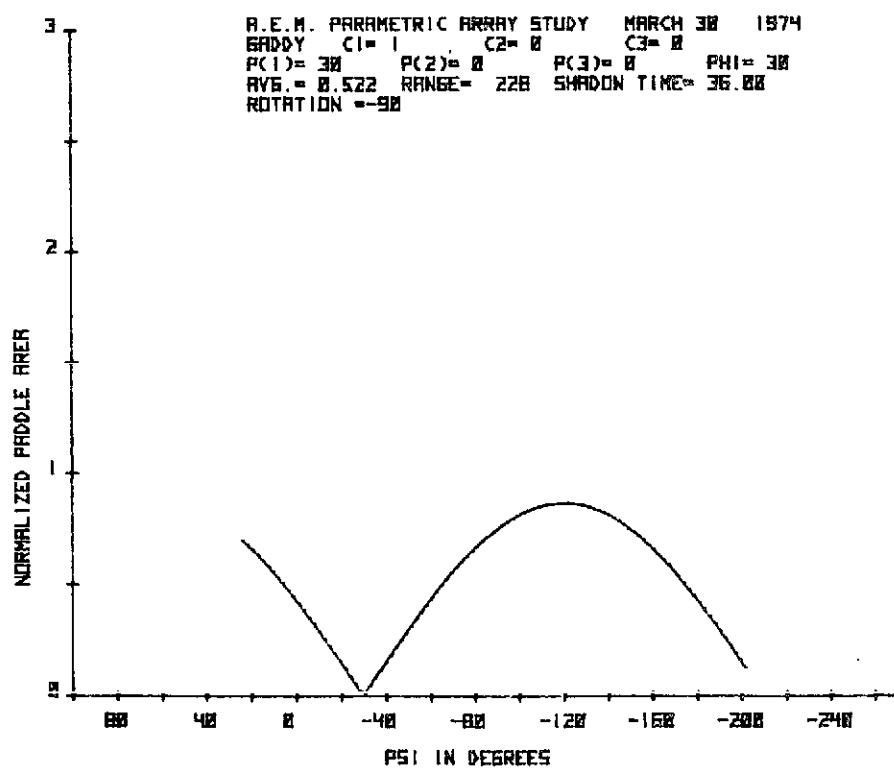
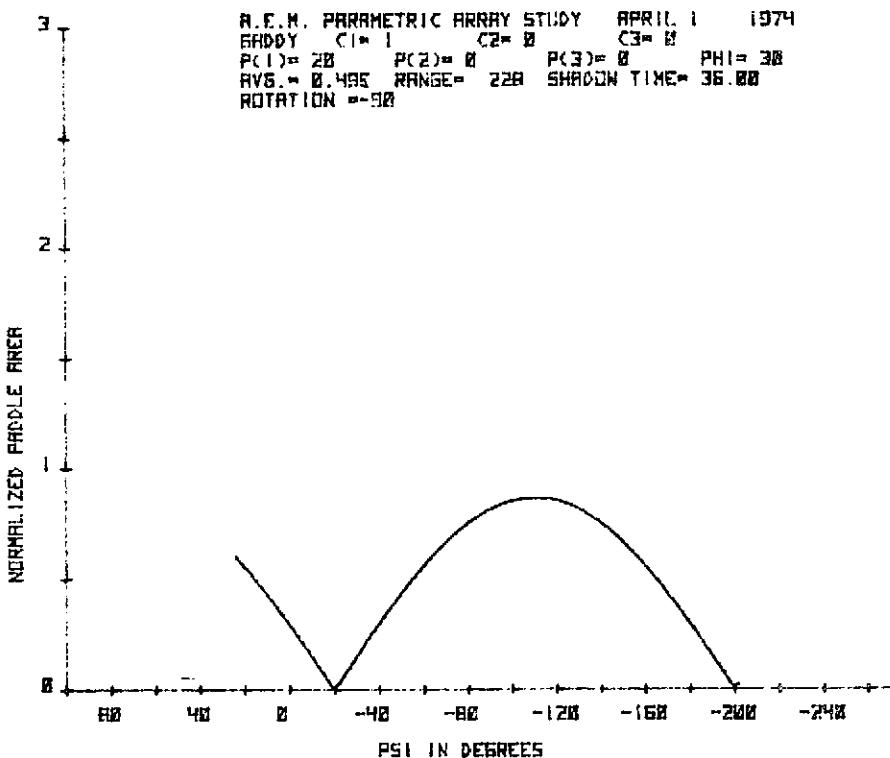
BETA ANGLE VS DAY NUMBER A.E.M.-1

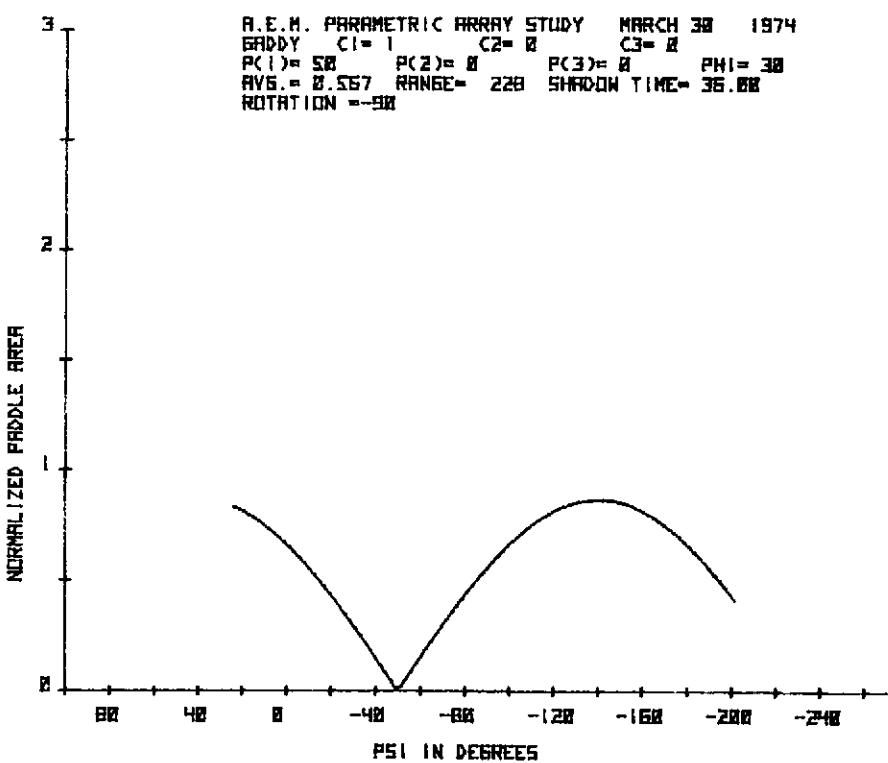
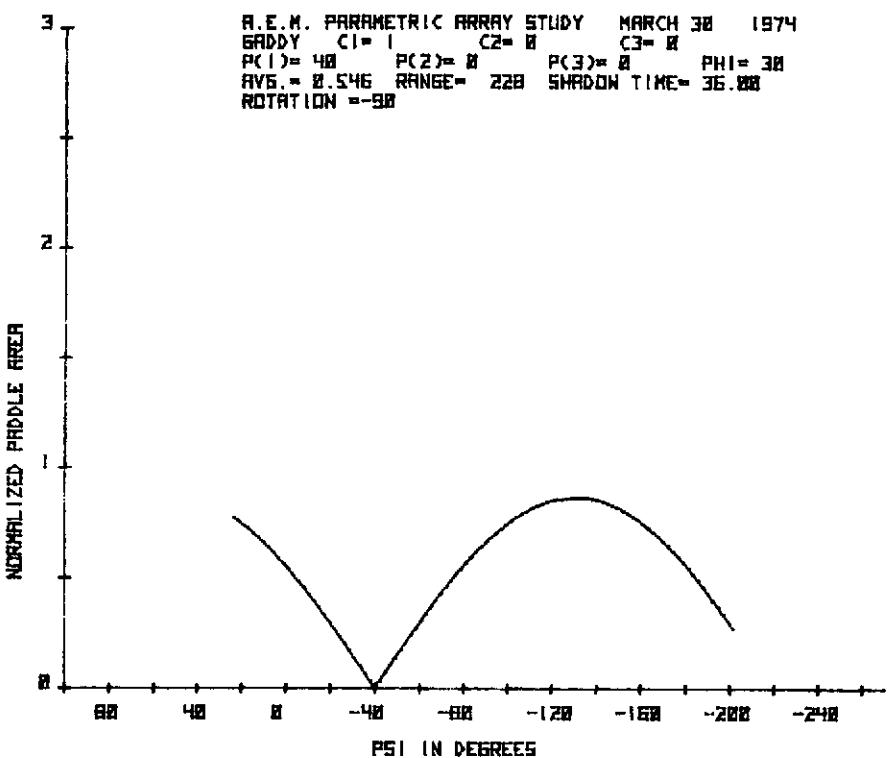


APPENDIX C



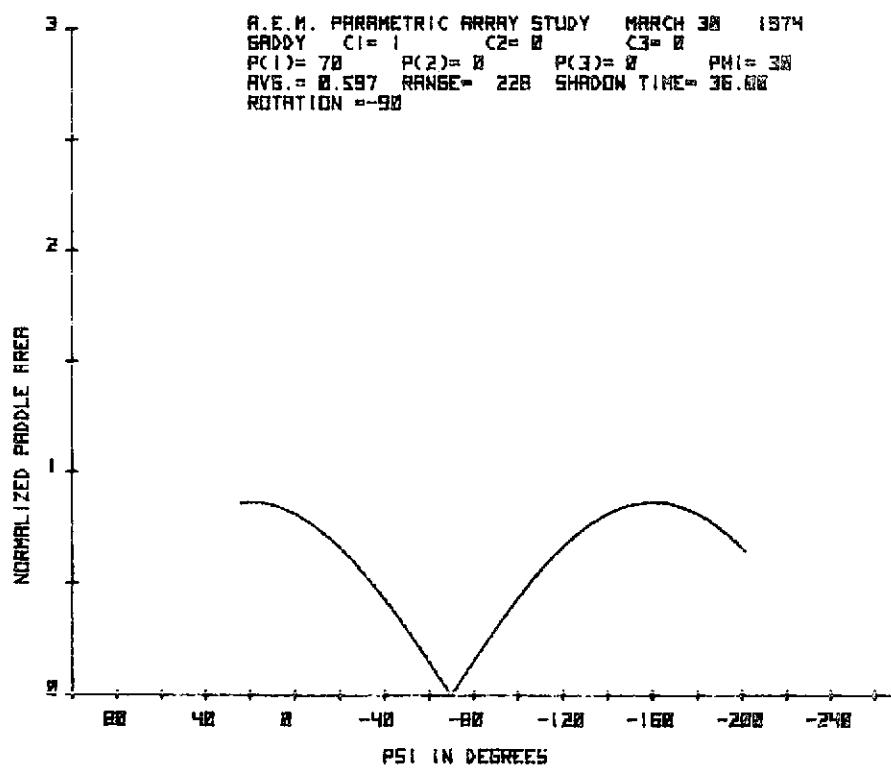
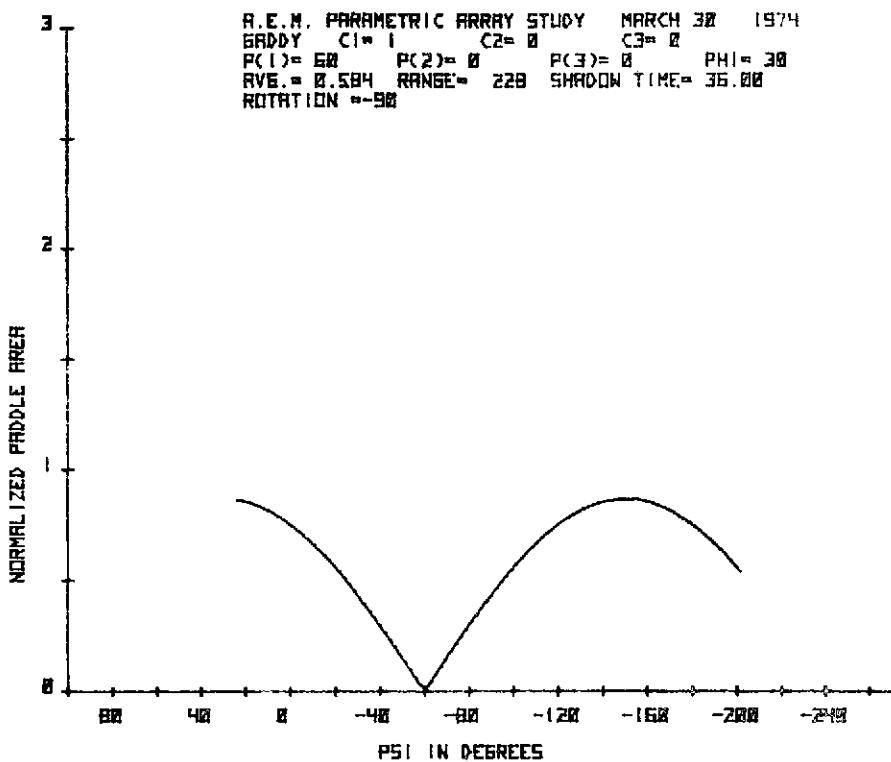
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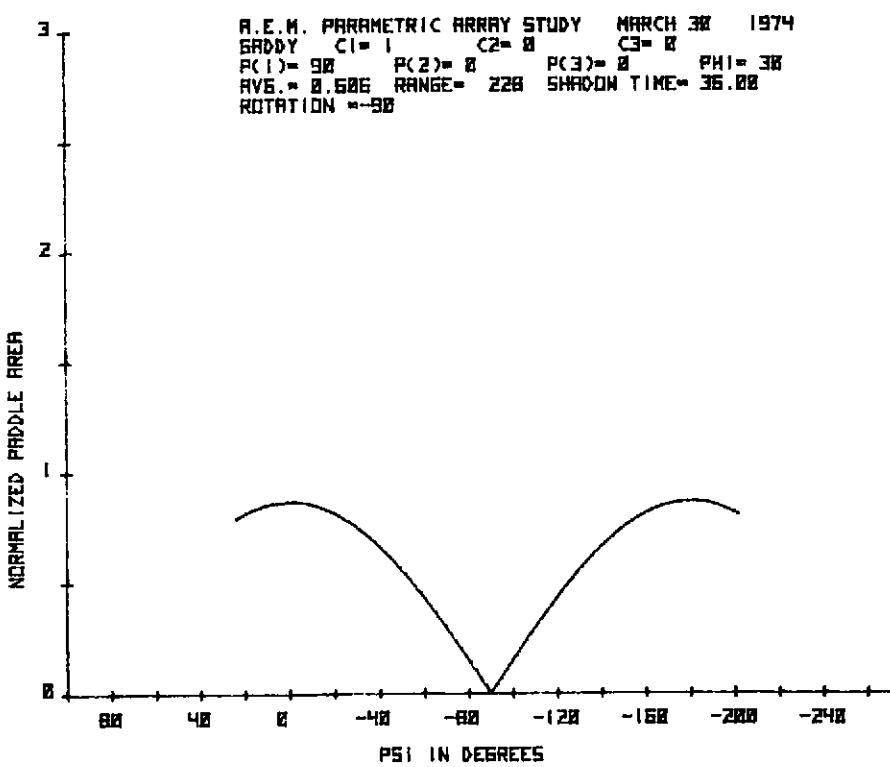
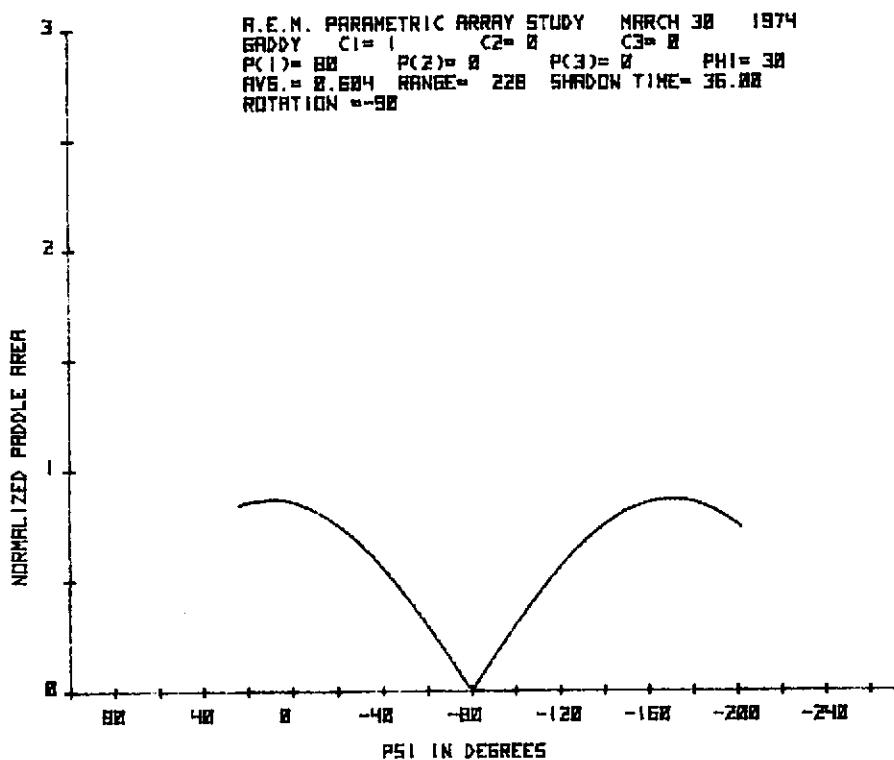


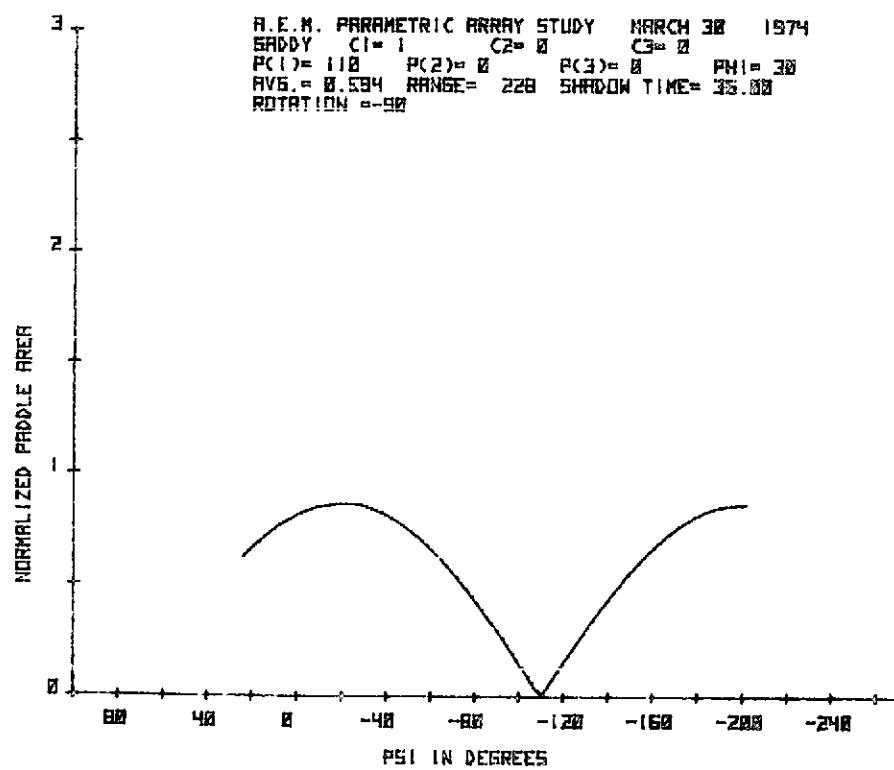
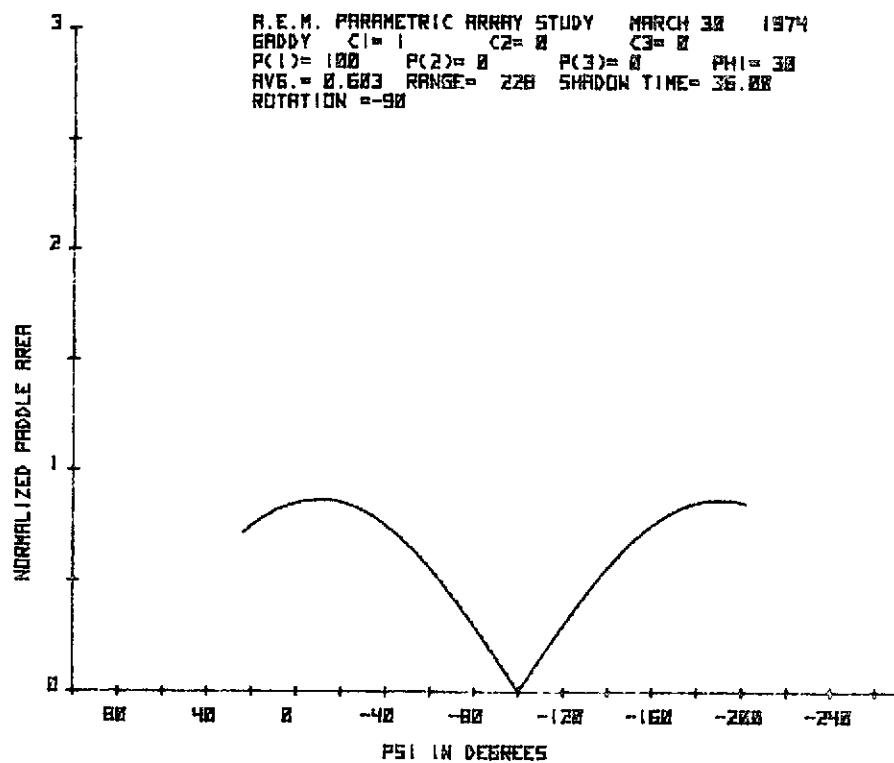


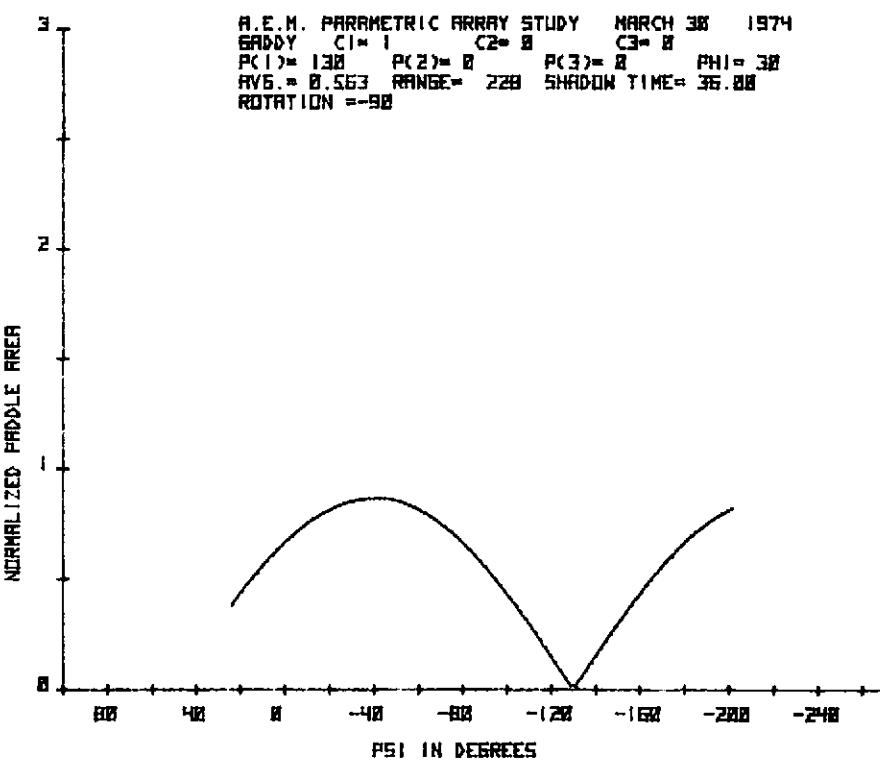
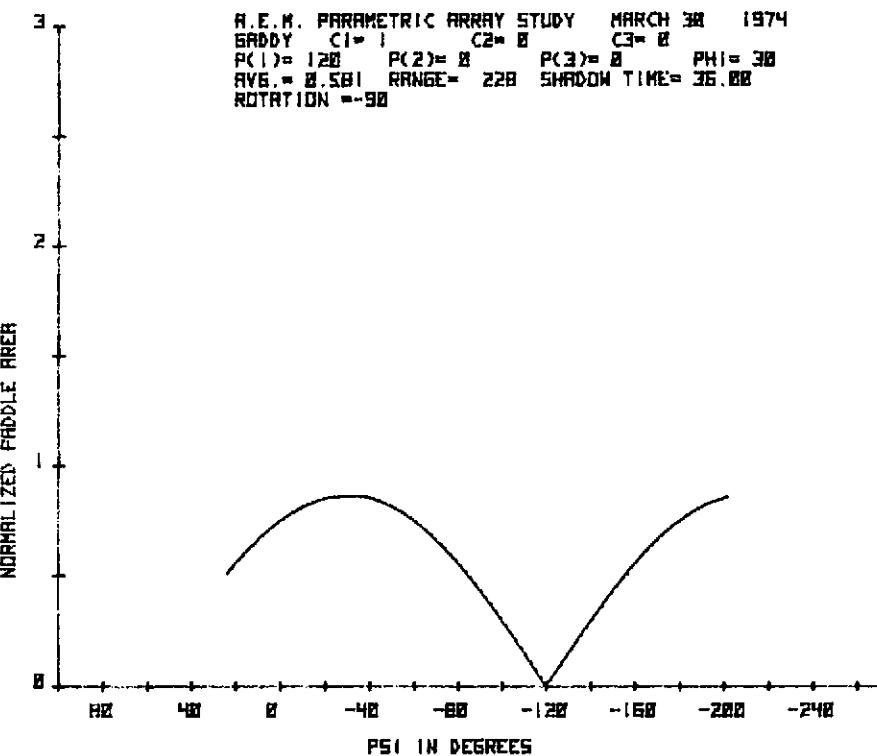
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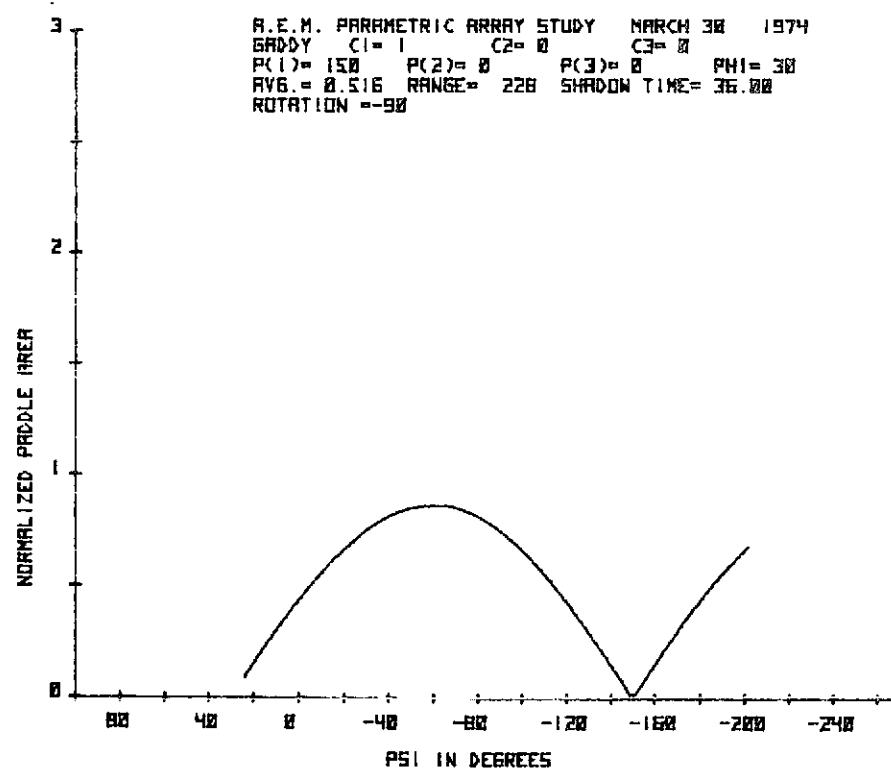
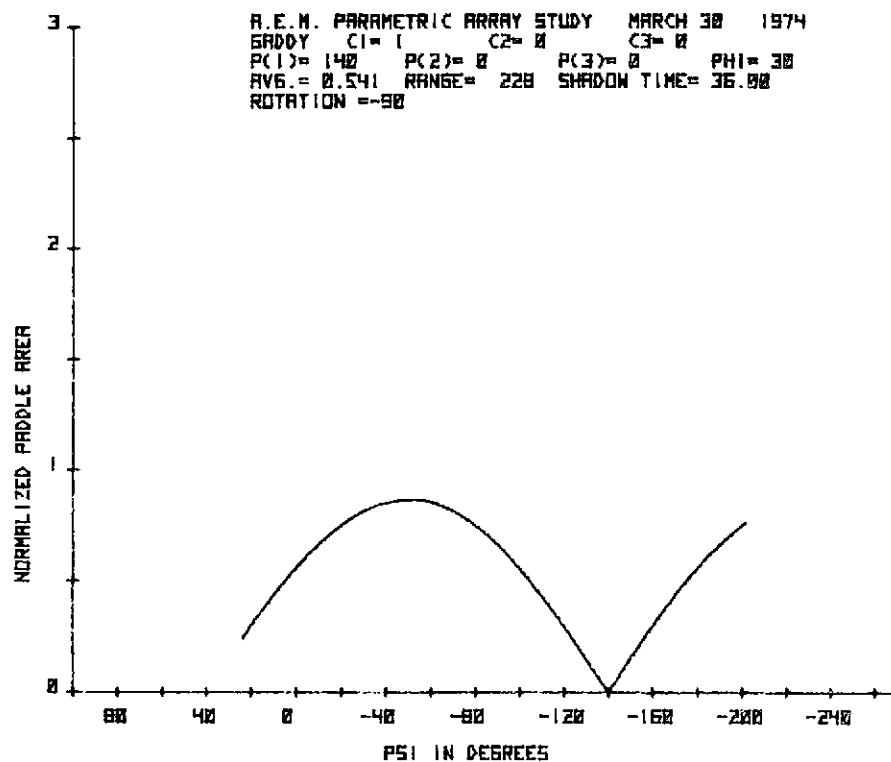


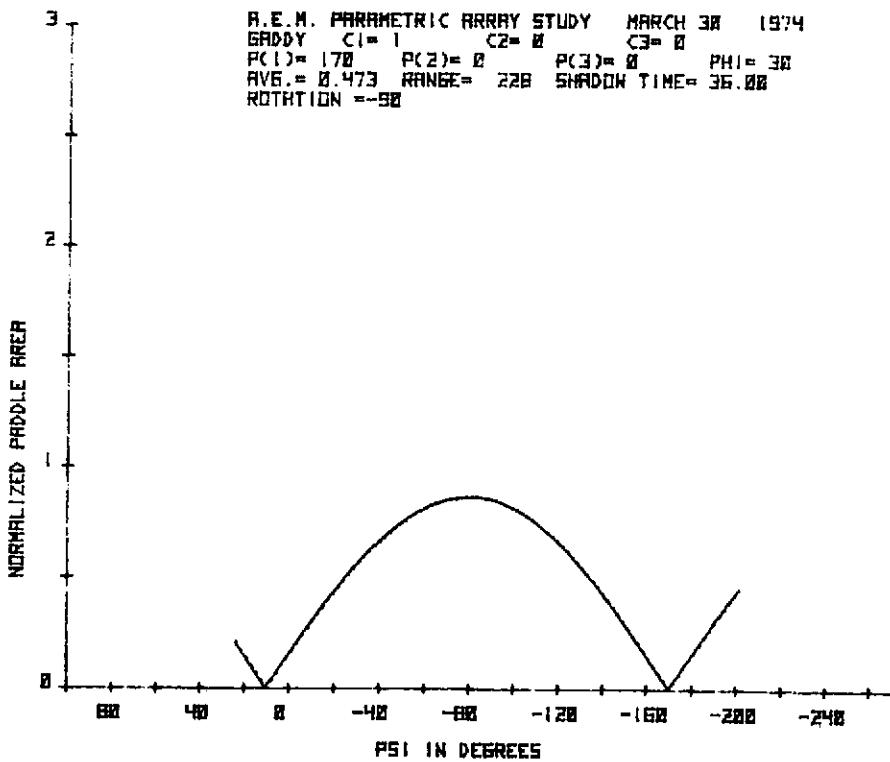
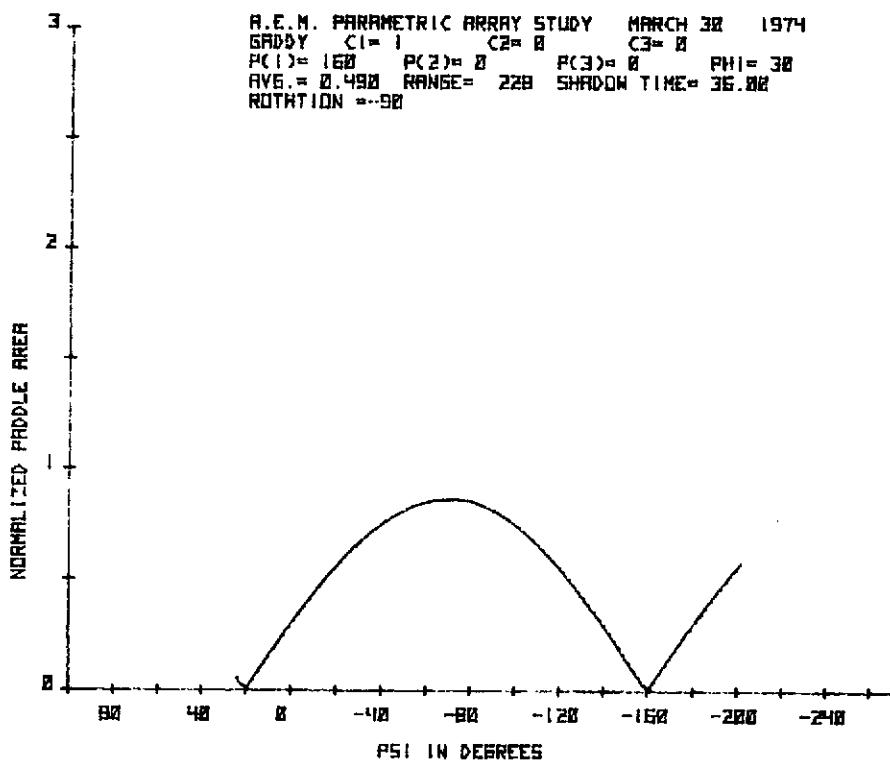


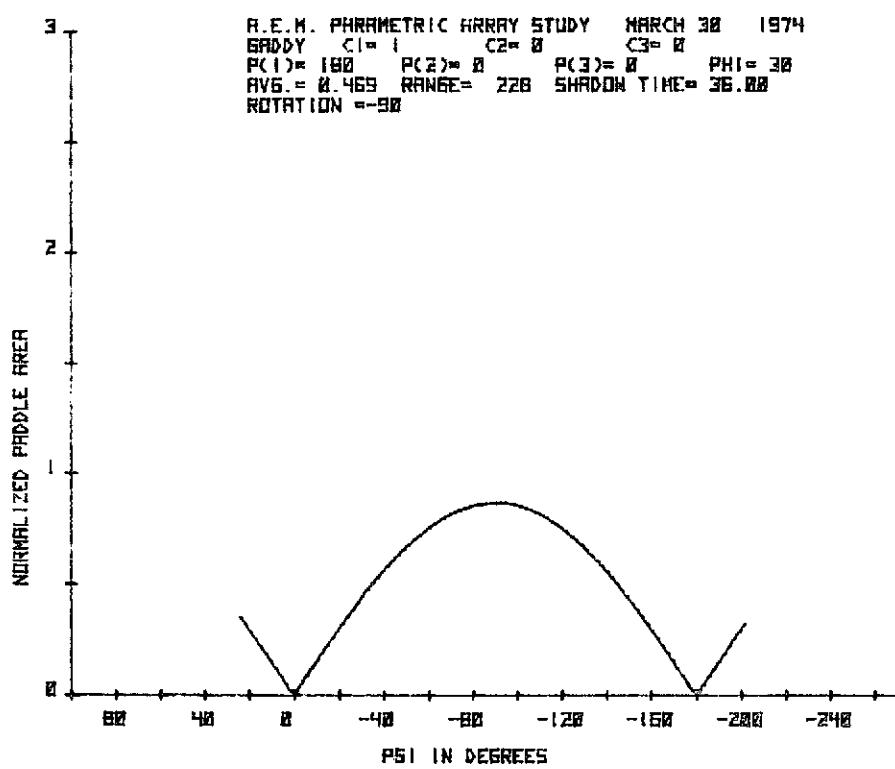


C-7

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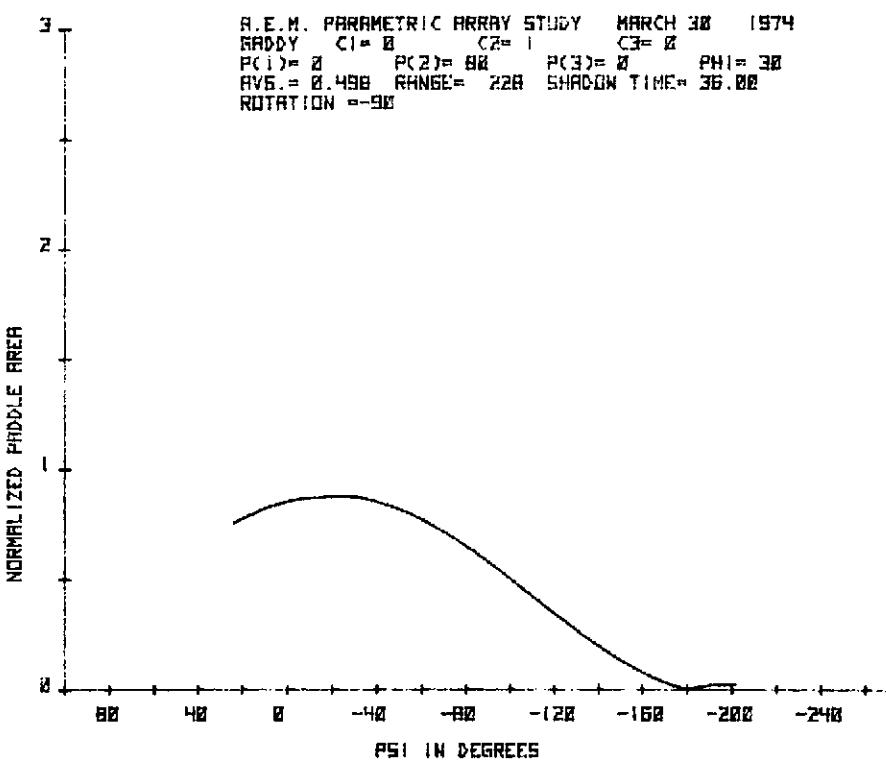
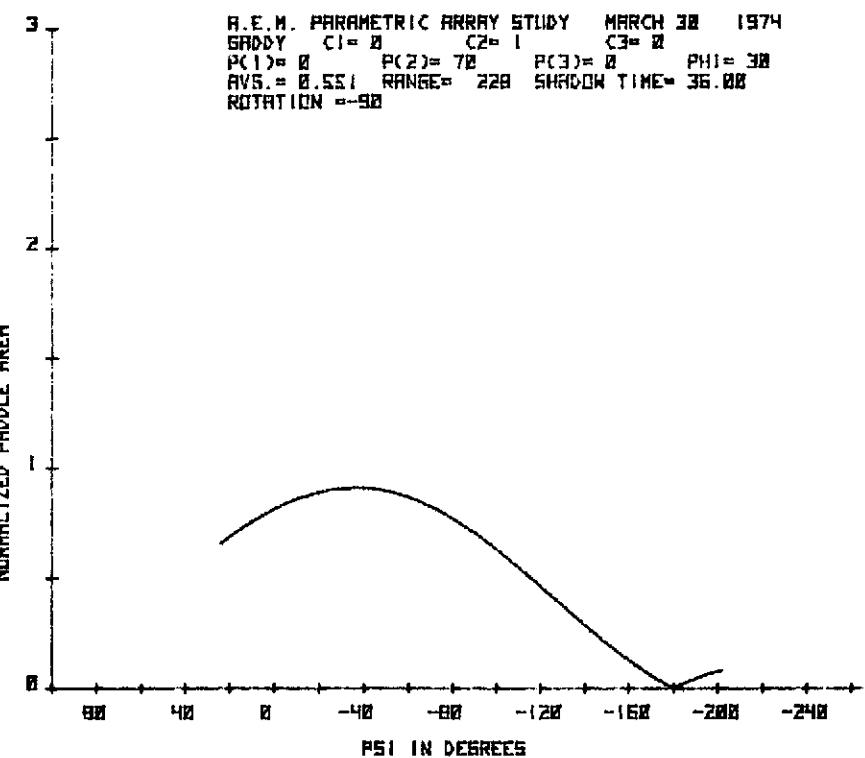




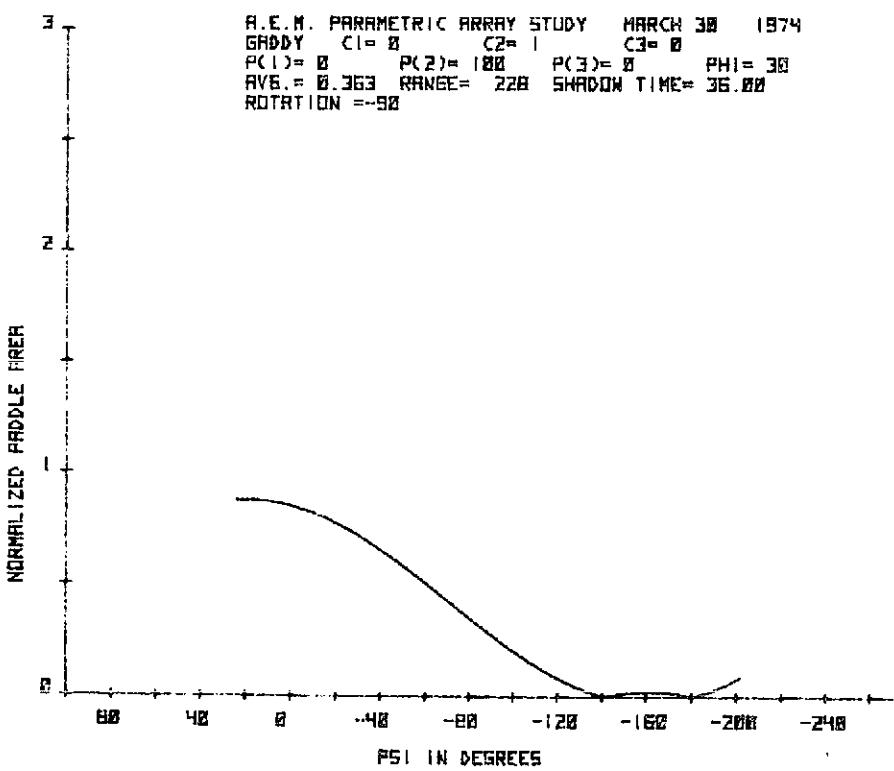
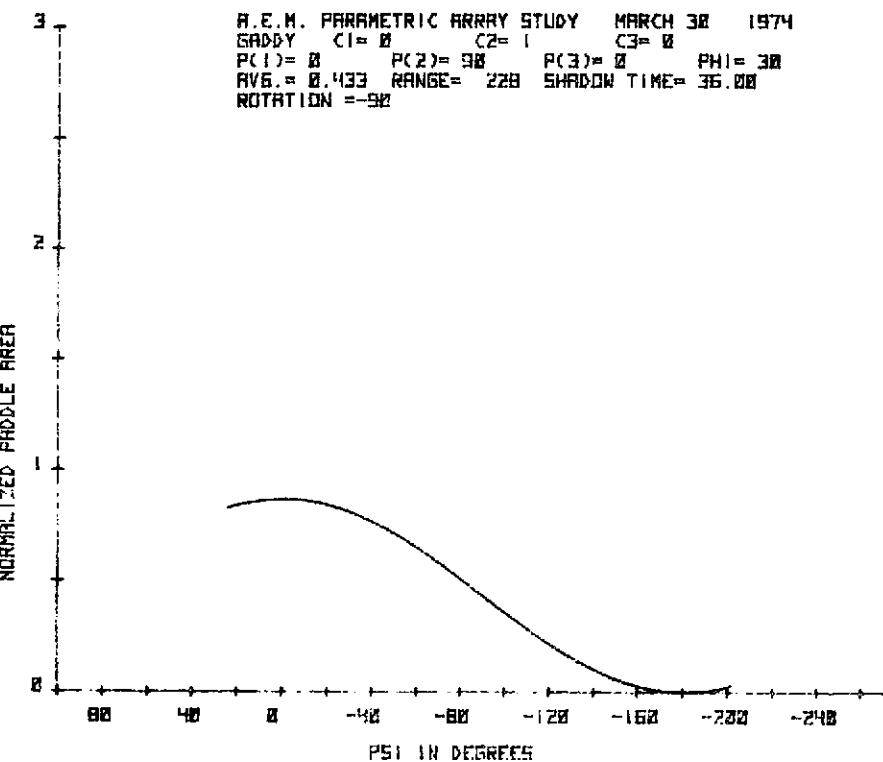
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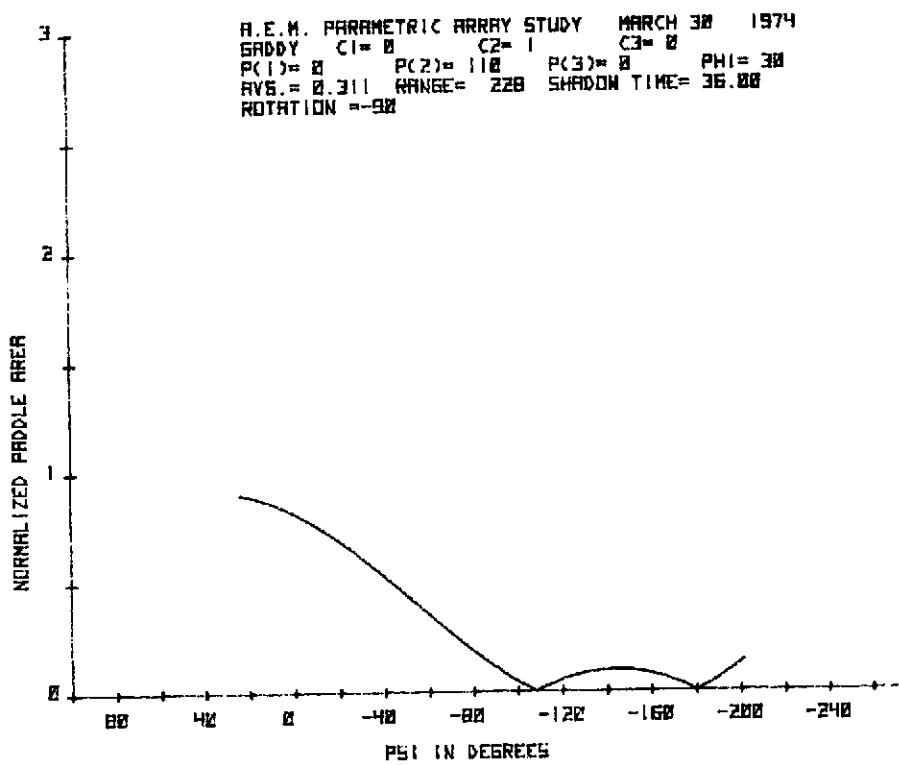
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APPENDIX D



D-1
30° <





D-3

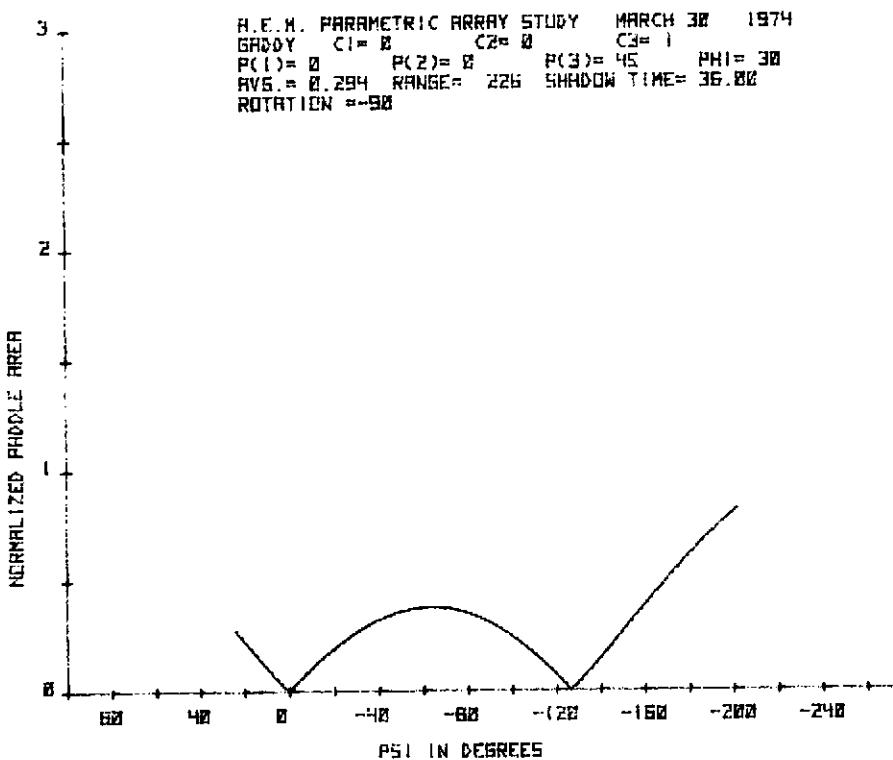
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APPENDIX E

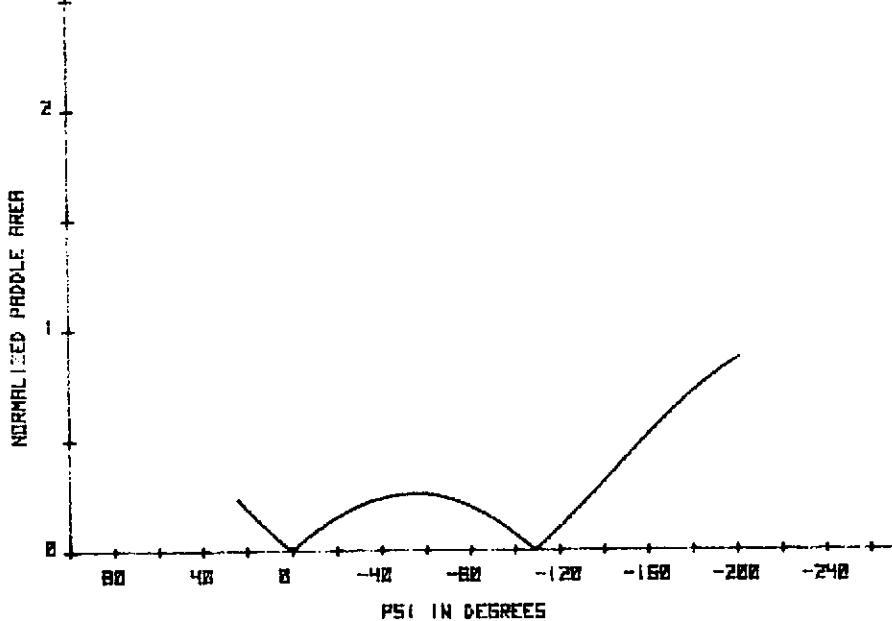
E-i

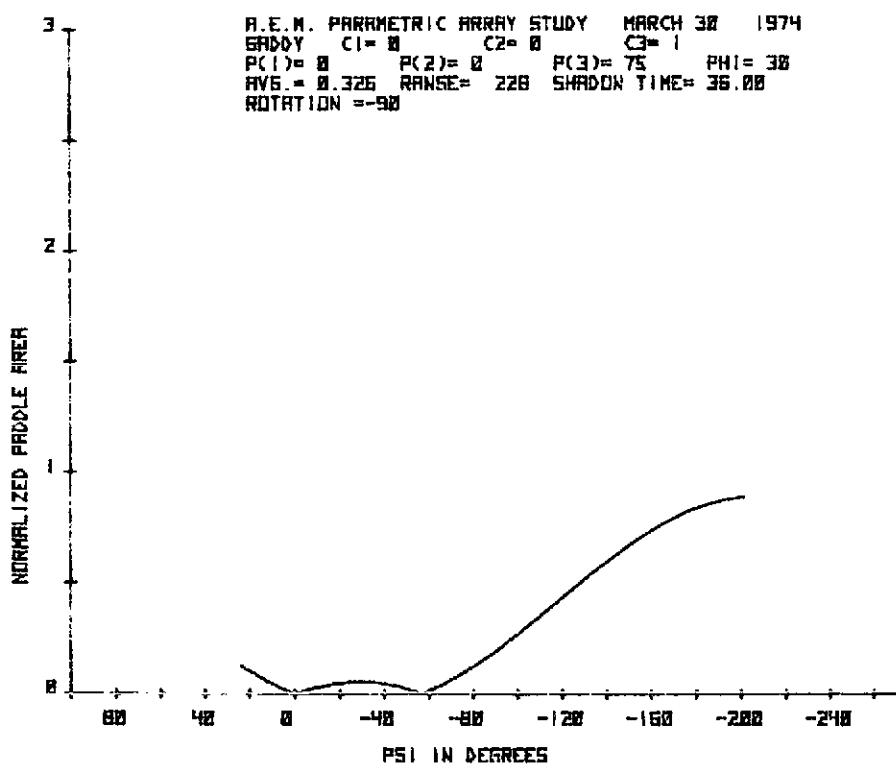
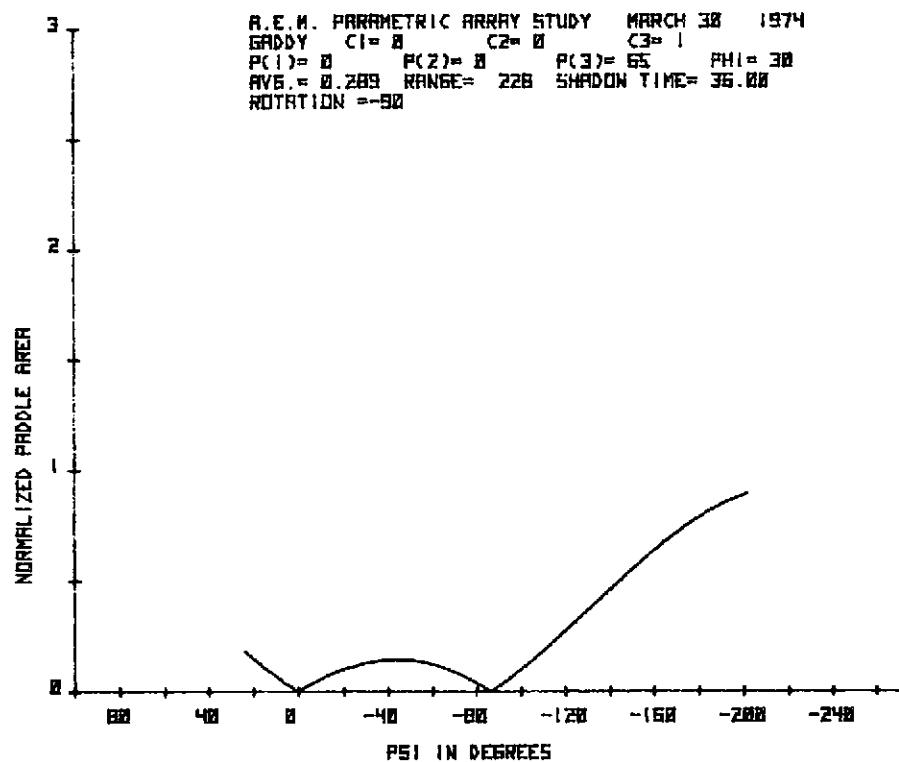
33<

H.E.M. PARAMETRIC ARRAY STUDY MARCH 30 1974
GRADDY $C_1 = 0$ $C_2 = 0$ $C_3 = 1$
 $P(1) = 0$ $P(2) = 0$ $P(3) = 45$ $\Phi H = 30$
AVG. = 0.294 RANGE = 226 SHADOW TIME = 36.00
ROTATION = -90

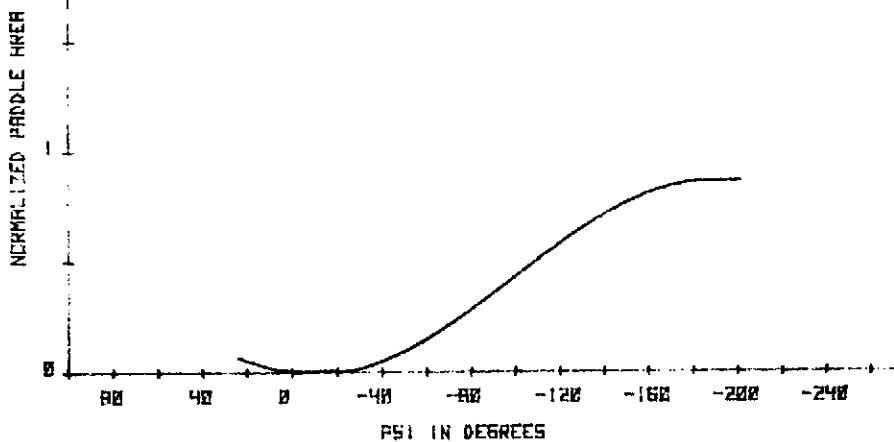


H.E.M. PARAMETRIC ARRAY STUDY MARCH 30 1974
GRADDY $C_1 = 0$ $C_2 = 0$ $C_3 = 1$
 $P(1) = 0$ $P(2) = 0$ $P(3) = 55$ $\Phi H = 30$
AVG. = 0.280 RANGE = 228 SHADOW TIME = 36.00
ROTATION = -90

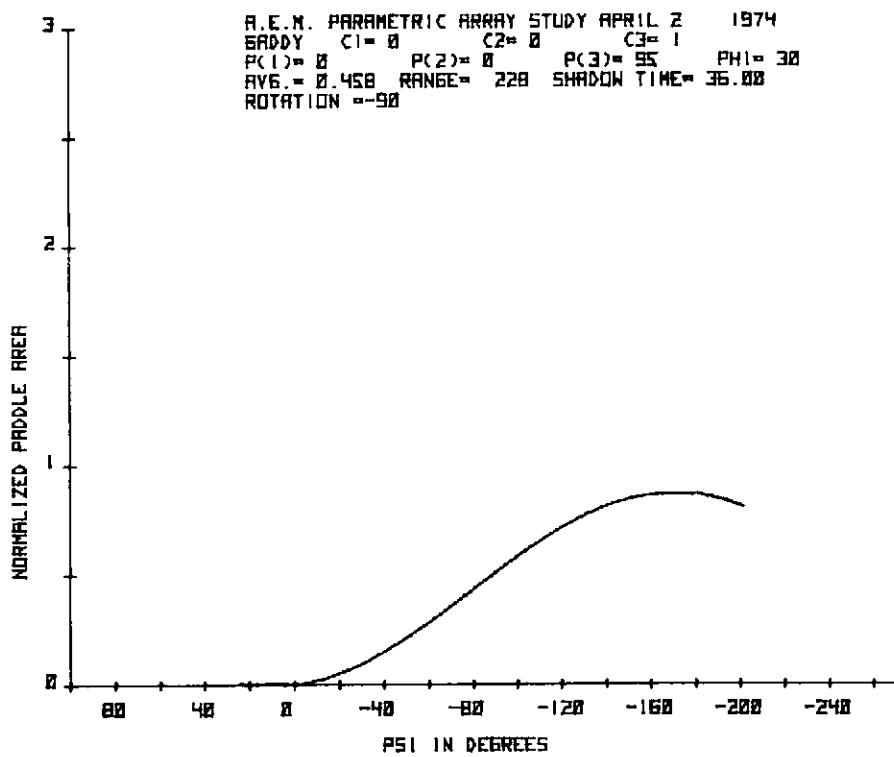


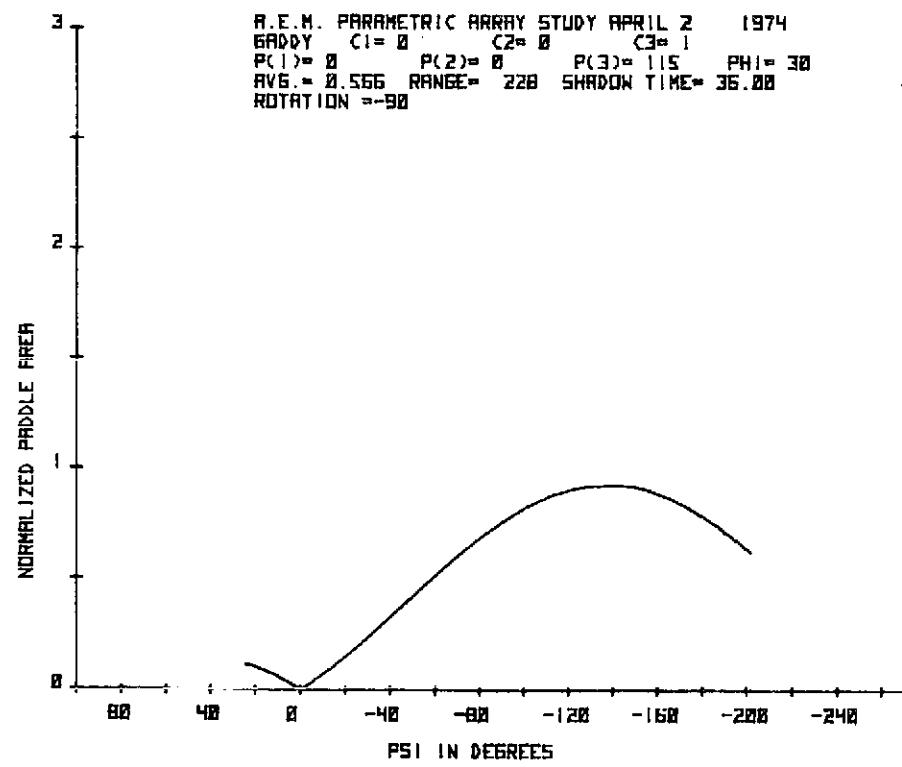
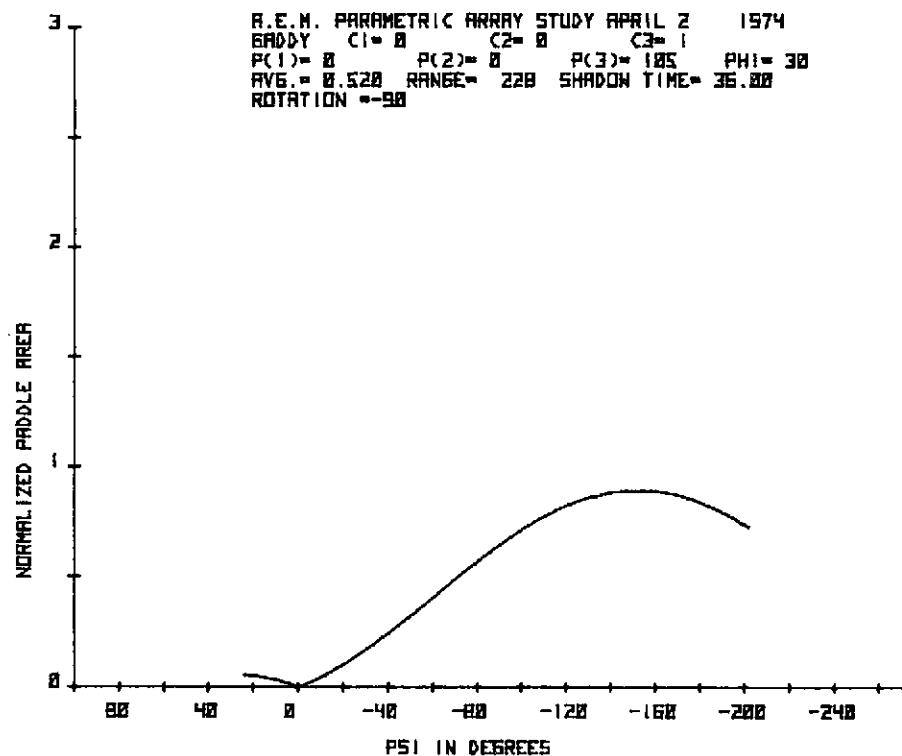


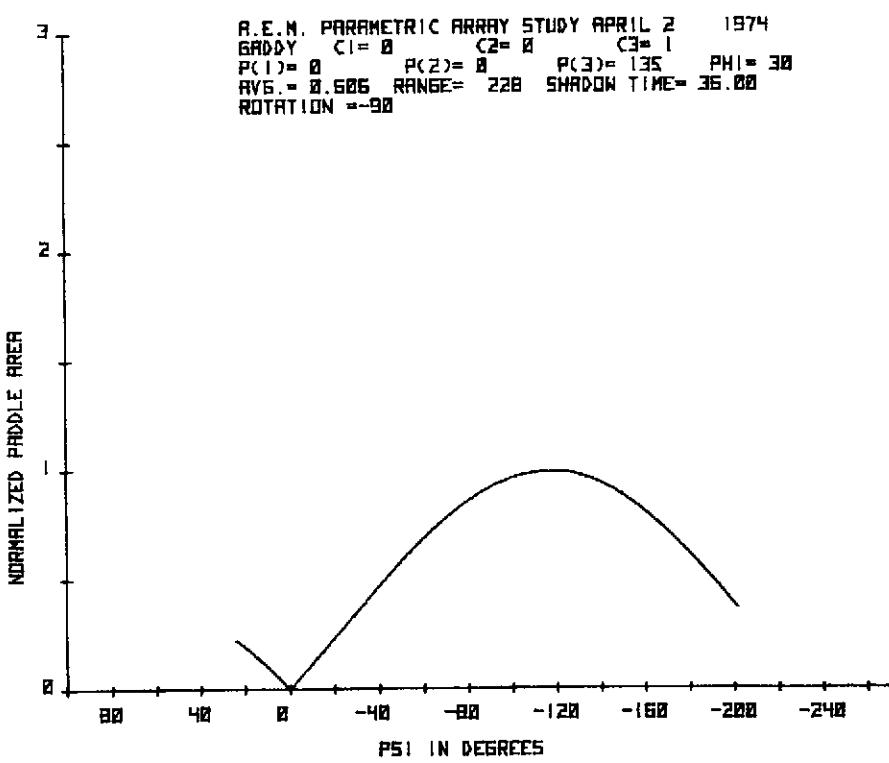
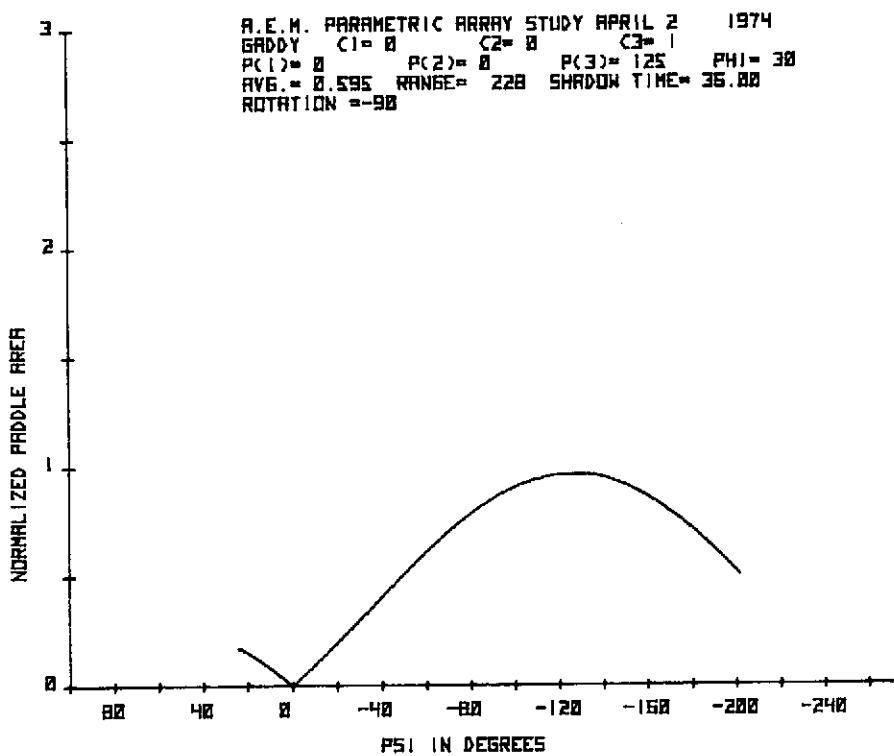
A.E.M. PARAMETRIC ARRAY STUDY APRIL 1 1974
GRADDY C1= 0 C2= 0 C3= 1
P(1)= 0 P(2)= 8 P(3)= 85 PH1= 30
AVG.= 0.350 RANGE= 228 SHADOW TIME= 36.00
ROTATION =-90



A.E.M. PARAMETRIC ARRAY STUDY APRIL 2 1974
GRADDY C1= 0 C2= 0 C3= 1
P(1)= 0 P(2)= 0 P(3)= 95 PH1= 30
AVG.= 0.458 RANGE= 228 SHADOW TIME= 36.00
ROTATION =-90



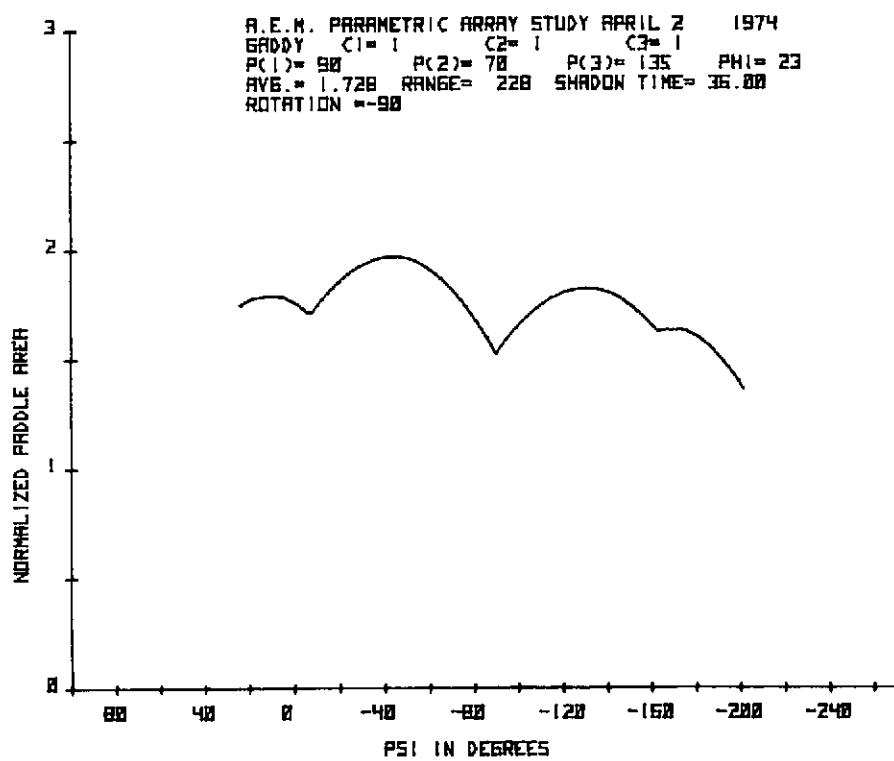
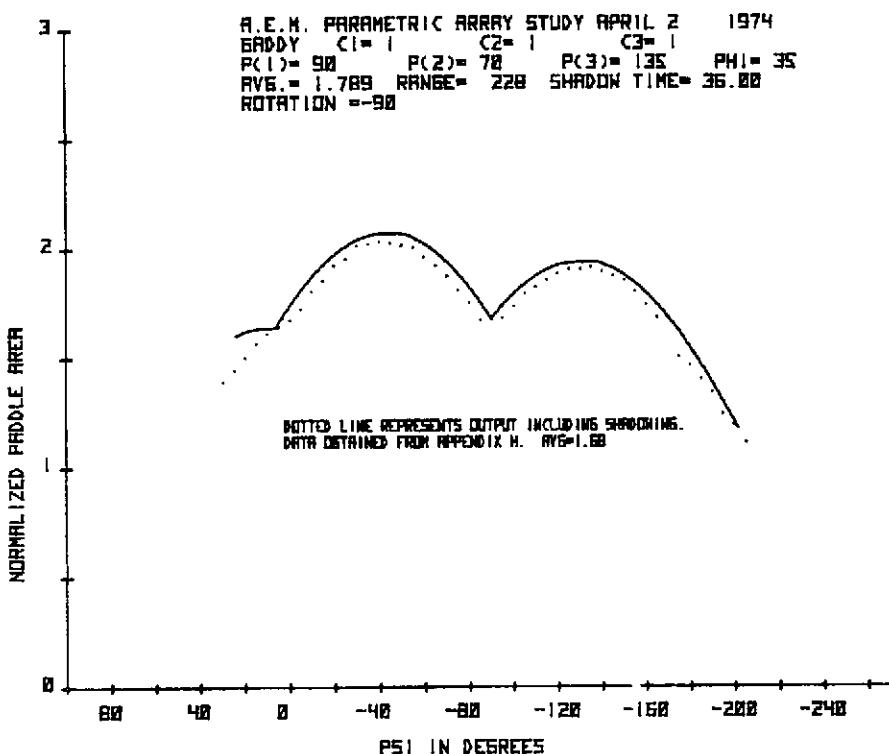


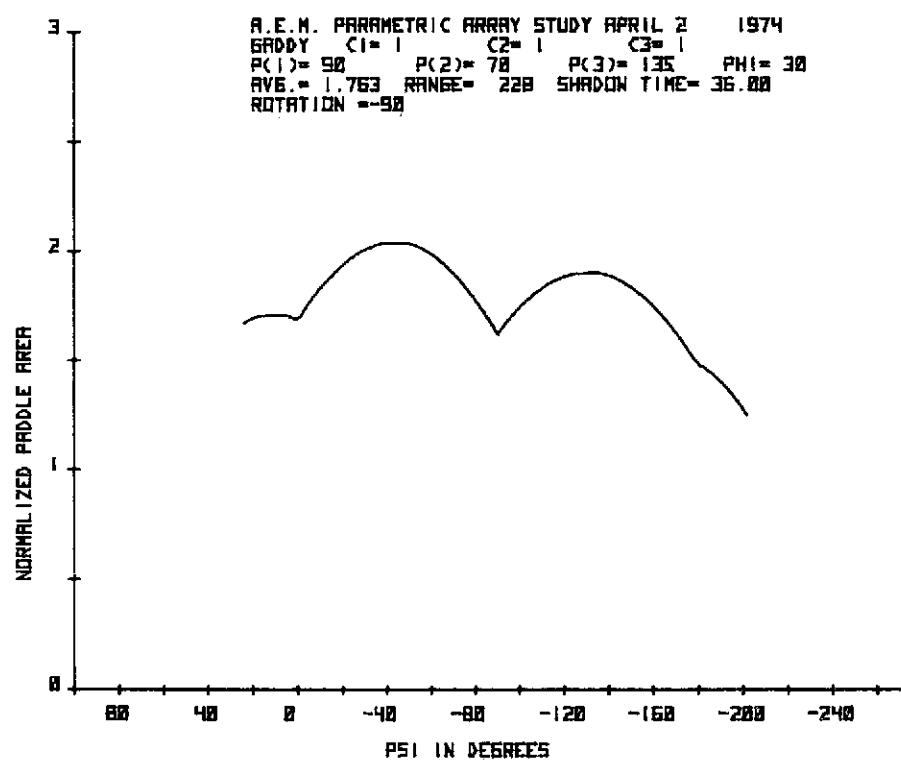


APPENDIX F

F-1

39<





APPENDIX G

Q-1

42<

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10 DEG
20 S=50
30 SCALE -S*3.5,S*3.5,-S*5,S*5
40 DIM A[3,3],B[3],C[3],D[3,3],E[3],F[3],G[3]
50 DISP "ENTER P(1),P(2),P(3),PHI,PSI";
60 INPUT P[1],P[2],P[3],P1,S1
70 W=0
80 D[1,1]=COS S1*COS P1
90 D[1,2]=-SIN S1*COS P1
100 D[1,3]=SIN P1
110 D[2,1]=COS S1*SIN P1
120 D[2,2]=-SIN S1*SIN P1
130 D[2,3]=-COS P1
140 D[3,1]=SIN S1
150 D[3,2]=+COS S1
160 D[3,3]=0
170 FOR P=R TO (240+R) STEP 120
180 T=P[1+(P-R)/120]
190 A[1,1]=COS P
200 A[1,2]=-COS T*SIN P
210 A[1,3]=SIN T*SIN P
220 A[2,1]=SIN P
230 A[2,2]=COS T*COS P
240 A[2,3]=-SIN T*COS P
250 A[3,1]=0
260 A[3,2]=SIN T
270 A[3,3]=COS T
280 FOR J=0 TO 2
290 FOR I=1 TO 7
300 MAT READ B
310 B[1]=B[1]+J*35.79
320 MAT C=A*B
330 G[1]=C[1]
340 G[2]=+C[3]
350 G[3]=-C[2]
360 MAT E=D*G
370 PLOT E[2],E[3],-2
380 IF J#2 THEN 580
390 IF I=3 THEN 430
400 IF I=4 THEN 460
410 IF I=5 THEN 490
420 GOTO 580
430 X1=E[2]
440 Y1=E[3]
450 GOTO 580
460 X2=E[2]
470 Y2=E[3]
480 GOTO 580
490 X3=E[2]
500 Y3=E[3]

```

```

510 L1=(ABS((X2-X1)↑2+(Y2-Y1)↑2))↑0.5
520 L2=(ABS((X3-X2)↑2+(Y3-Y2)↑2))↑0.5
530 R1=ATN((Y3-Y2)/(X3-X2))
540 R2=ATN((Y2-Y1)/(X2-X1))
550 L2=L2*SIN(R2-R1)
560 R=ABS((L1*L2)/509.278)
570 W=W+A
580 NEXT I
590 PEN
600 RESTORE
610 NEXT J
620 LABEL (630,1.5,1.7,0,10/7)A,(1+(P-R)/120)
630 FORMAT 4X,F6.3,X,F2.0
640 PEN
650 NEXT P
660 PLOT -S*3.5,-S*5,1
670 CPLOT 10,6
680 FIXED 2
690 LABEL (*)"TOTAL ="!W
700 STANDARD
710 LABEL (*)"P(1)=""P[1];" P(2)=""P[2];" P(3)=""P[3]
720 LABEL (*)"PHI=""P1;" PSI=""S1;" R=""R
730 DATA 15.01,0,0,17.05,0,0,17.05,-7.7,0,50.12,-7.7,0,50.12,7.7,0
740 DATA 17.05,7.7,0,17.05,0,0
750 RESTORE 1000
760 FOR P=R TO (300+R) STEP 60
770 GOSUB 1320
780 FOR I=1 TO 5
790 IF S1<-180 OR S1>0 THEN 840
800 IF I#4 THEN 820
810 IF E[1]XX THEN 960
820 IF I#5 THEN 840
830 IF E[1]XX THEN 950
840 MAT READ B
850 MAT C=A*B
860 G[1]=C[1]
870 G[2]=+C[3]
880 G[3]=-C[2]
890 MAT E=D*G
900 IF I#1 THEN 930
910 X=E[1]
920 GOTO 940
930 PLOT E[2],E[3],-2
940 NEXT I
950 PEN
960 RESTORE 1000
970 NEXT P
980 PEN
990 RESTORE 1100
1000 DATA 0,0,-2.945,15.01,-8.7,-2.945,15.01,8.7,-2.945,15.01,8.7,-19.67

```

```

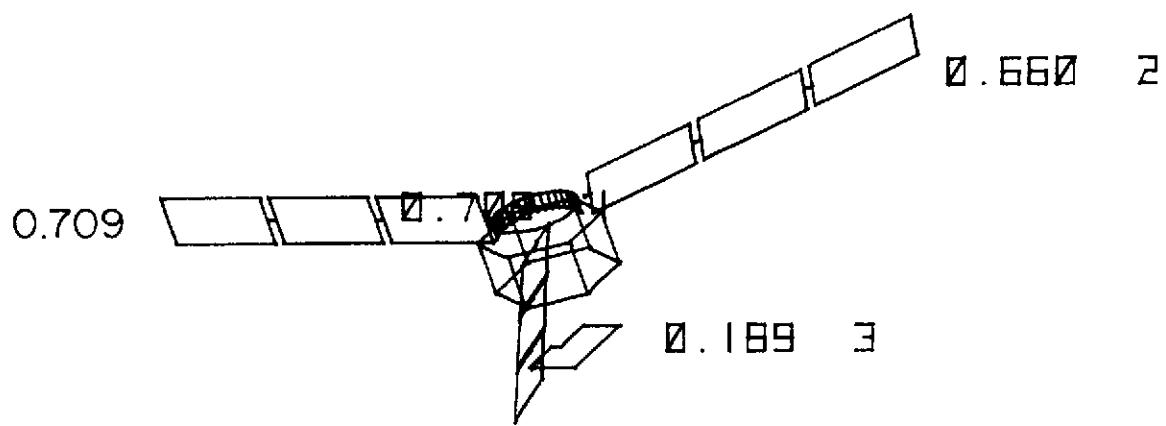
1010 DATA 15.01,-8.7,-19.67
1020 FOR P=0 TO 350 STEP 10
1030 GOSUB 1320
1040 FOR I=1 TO 4
1050 IF I#3 THEN 1070
1060 IF E[1]<0 THEN 1160
1070 MAT READ B
1080 MAT C=R*B
1090 G[1]=C[1]
1100 G[2]=+C[3]
1110 G[3]=-C[2]
1120 MAT E=D*G
1130 PLOT E[2],E[3],-2
1140 NEXT I
1150 PEN
1160 RESTORE 1180
1170 NEXT P
1180 DATA 12.125,-1.057,2.33,12.125,1.057,2.33,12.125,1.057,-2.945
1190 DATA 12.125,-1.057,-2.945
1200 PEN
1210 IF S1>-180 AND S1<0 THEN 1300
1220 RESTORE 1280
1230 FOR I=1 TO 7
1240 MAT READ B
1250 MAT E=D*B
1260 PLOT E[2],E[3],-2
1270 NEXT I
1280 DATA 11.2,-42.32,-5.5,-12.6,-42.32,-5.5,-12.6,-42.32,9.8,-0.9,-42.32,9.8
1290 DATA -0.9,-42.32,6.6,11.2,-42.32,6.6,11.2,-42.32,-5.5
1300 PLOT S*3.5,S*5,1
1310 END
1320 A[1,1]=R[2,2]=COSP
1330 A[1,2]=-SINF
1340 A[1,3]=A[2,3]=A[3,2]=0
1350 A[2,1]=SINF
1360 A[3,3]=1
1370 RETURN

```

APPENDIX H

H-i

46<

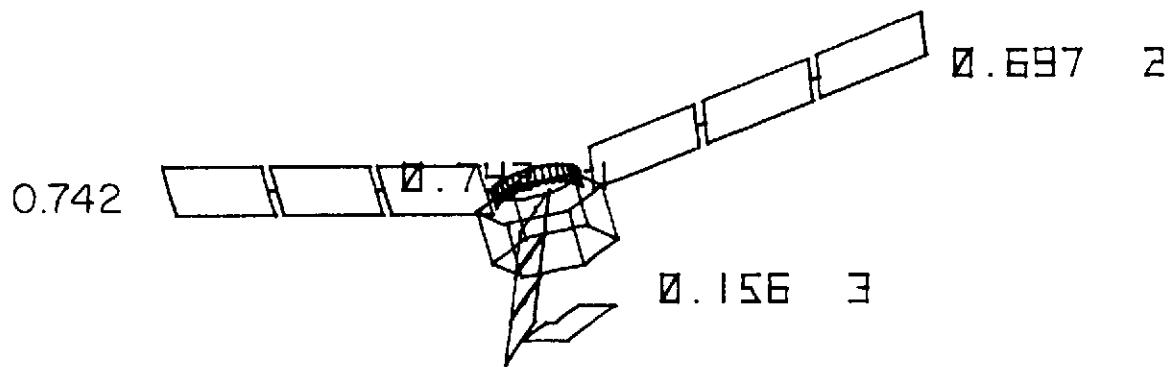


TOTAL = 1.56

P(1) = 90 P(2) = 70 P(3) = 135

PHI = 35 PSI = 30 R = -90

TOTAL AFTER SHADOWING IS $1.56 - (11/12) (.189) = 1.39$



TOTAL = 1.60

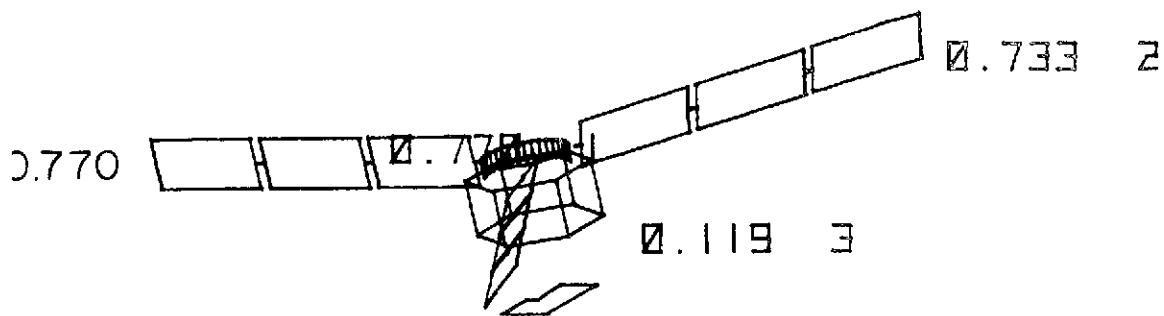
P(1) = 90 P(2) = 70 P(3) = 135

PHI = 35 PSI = 25 R = -90

TOTAL AFTER SHADOWING IS $1.60 - 0.156 = 1.44$

H-1

47<



TOTAL = 1.62

P(1)= 90

PH1= 35

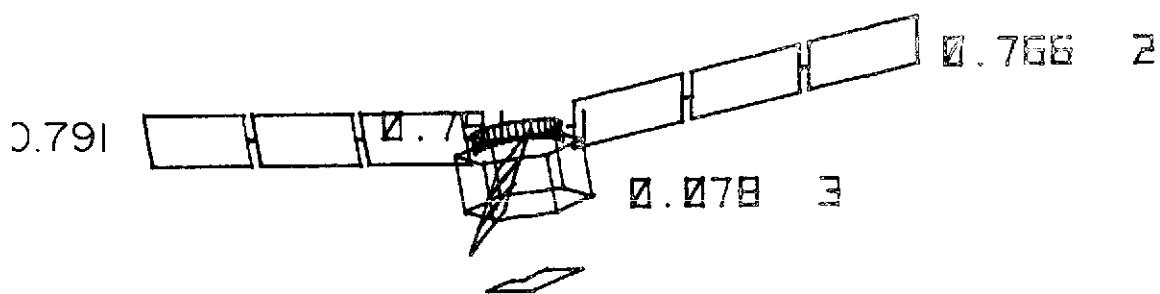
P(2)= 70

PS1= 20

P(3)= 135

R=-90

TOTAL AFTER SHADOWING IS 1.62 - 0.119 = 1.50



TOTAL = 1.64

P(1)= 90

PH1= 35

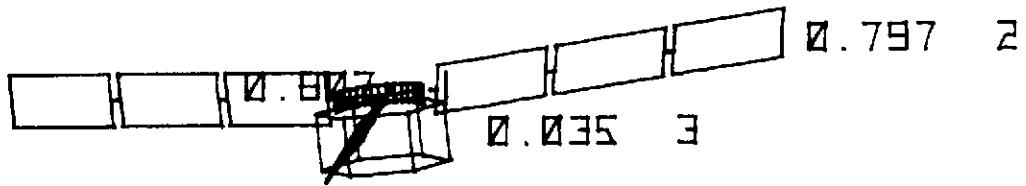
P(2)= 70

PS1= 15

P(3)= 135

R=-90

TOTAL AFTER SHADOWING IS 1.64 - 0.078 = 1.56



TOTAL = 1.64

P(1) = 90

PH1 = 35

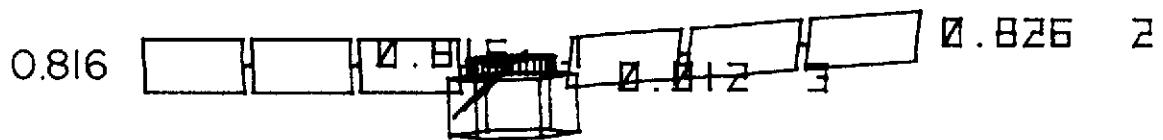
P(2) = 70

PS1 = 10

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.64 - 0.035 = 1.61



TOTAL = 1.65

P(1) = 90

PH1 = 35

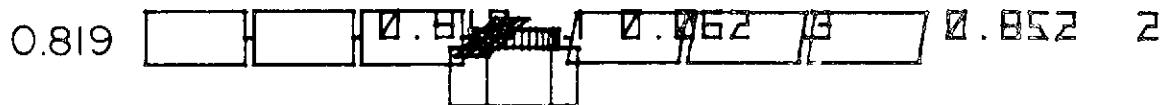
P(2) = 70

PS1 = 5

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.65 - 0.012 = 1.64



TOTAL = 1.73

P(1) = 90

PH1 = 35

P(2) = 70

PS1 = 0

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.73 - 0.062 = 1.67



TOTAL = 1.80

P(1) = 90

PH1 = 35

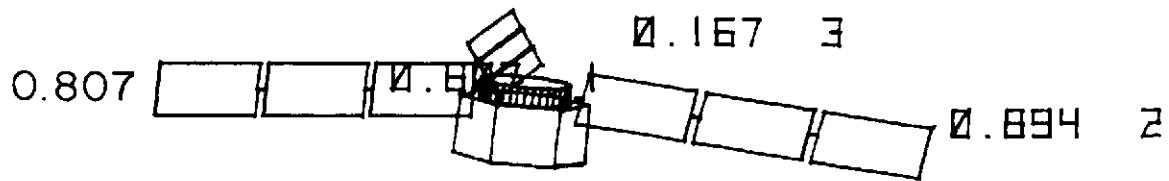
P(2) = 70

PS1 = -5

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.80 - (9/12) (.113) = 1.72



$$\text{TOTAL} = 1.87$$

$$P(1) = 90$$

$$PH1 = 35$$

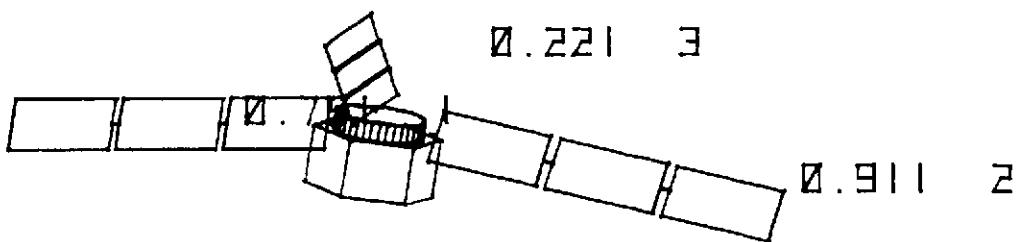
$$P(2) = 70$$

$$PS1 = -10$$

$$P(3) = 135$$

$$R = -90$$

TOTAL AFTER SHADOWING IS $1.87 - (5/12) (.167) = 1.80$



$$\text{TOTAL} = 1.92$$

$$P(1) = 90$$

$$PH1 = 35$$

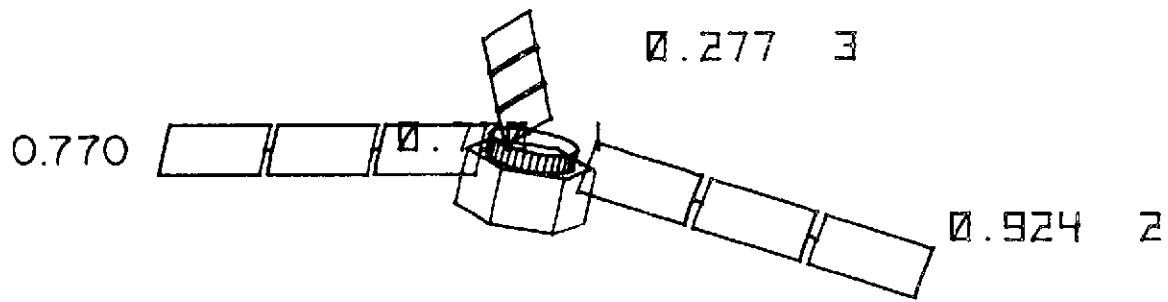
$$P(2) = 70$$

$$PS1 = -15$$

$$P(3) = 135$$

$$R = -90$$

TOTAL AFTER SHADOWING IS $1.92 - (4/12) (.221) = 1.85$



TOTAL = 1.97

P(1) = 90

PHI = 35

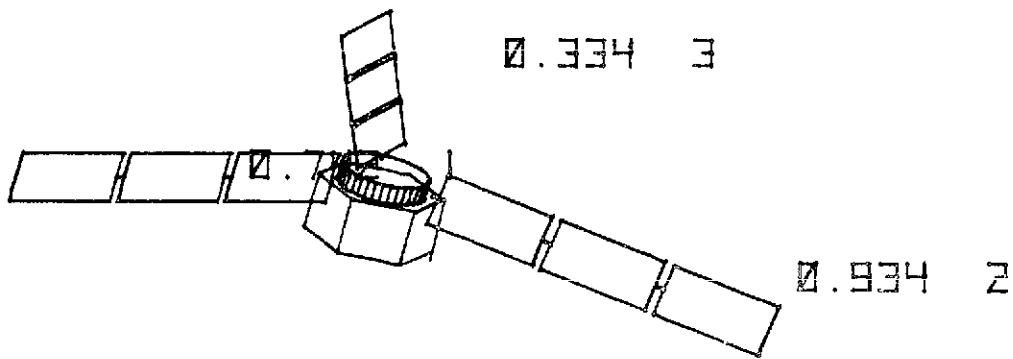
P(2) = 70

PSI = -20

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.97 - (2/12) (.277) = 1.92



TOTAL = 2.01

P(1) = 90

PHI = 35

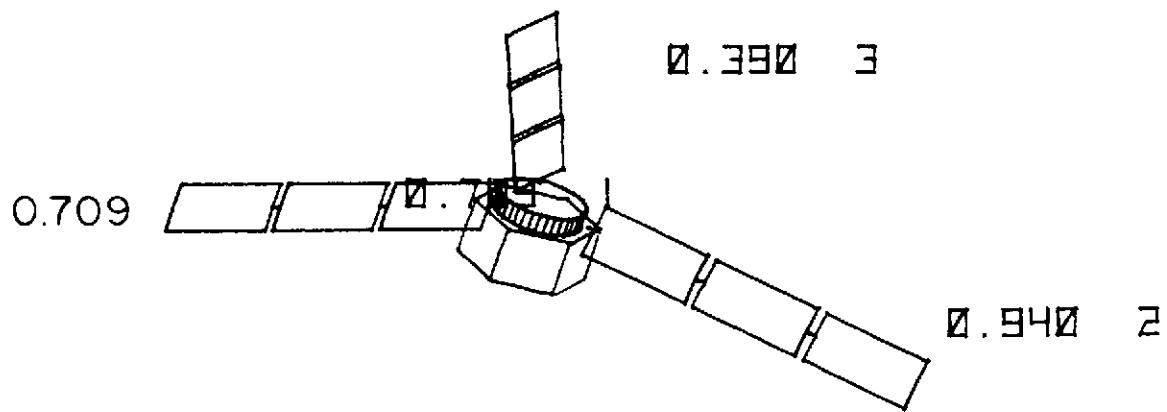
P(2) = 70

PSI = -25

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 2.01 - (2/12) (.334) = 1.95



TOTAL = 2.04

P(1)= 90

PH1= 35

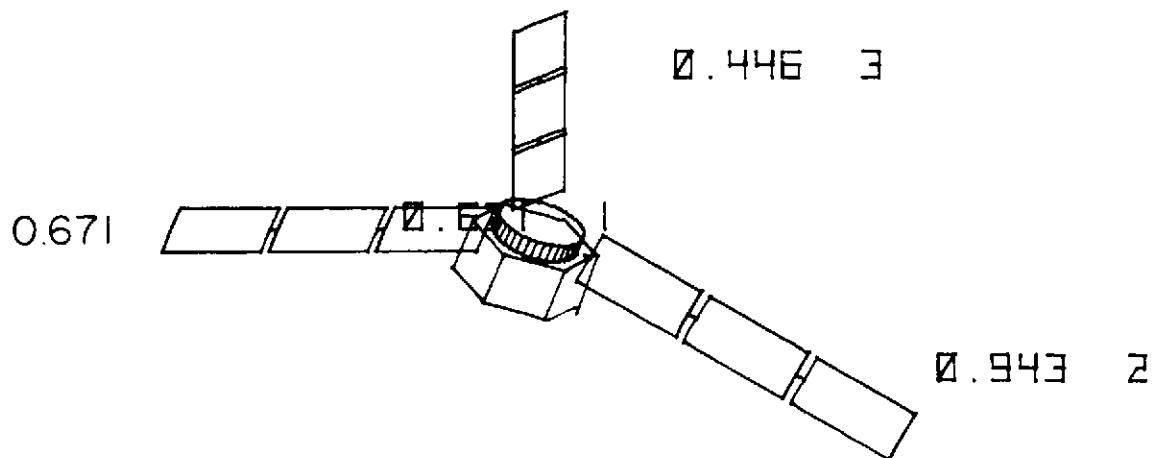
P(2)= 70

PS1=-30

P(3)= 135

R=-90

TOTAL AFTER SHADOWING IS 2.04 - (1/12) (.390) = 2.01



TOTAL = 2.06

P(1)= 90

PH1= 35

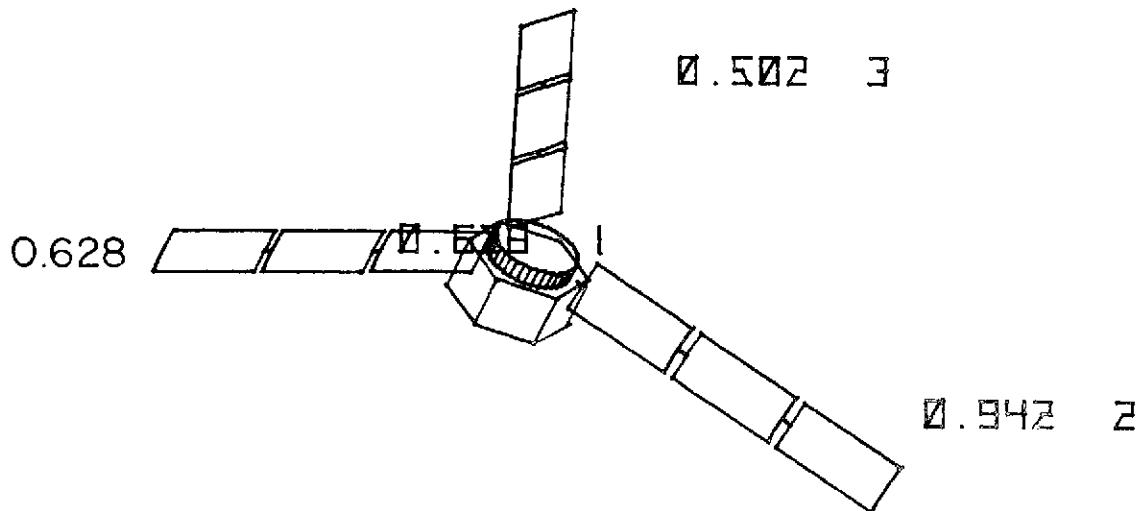
P(2)= 70

PS1=-35

P(3)= 135

R=-90

TOTAL AFTER SHADOWING IS 2.06 - (1/12) (.446) = 2.02



TOTAL = 2.07

P(1) = 90

PH1 = 35

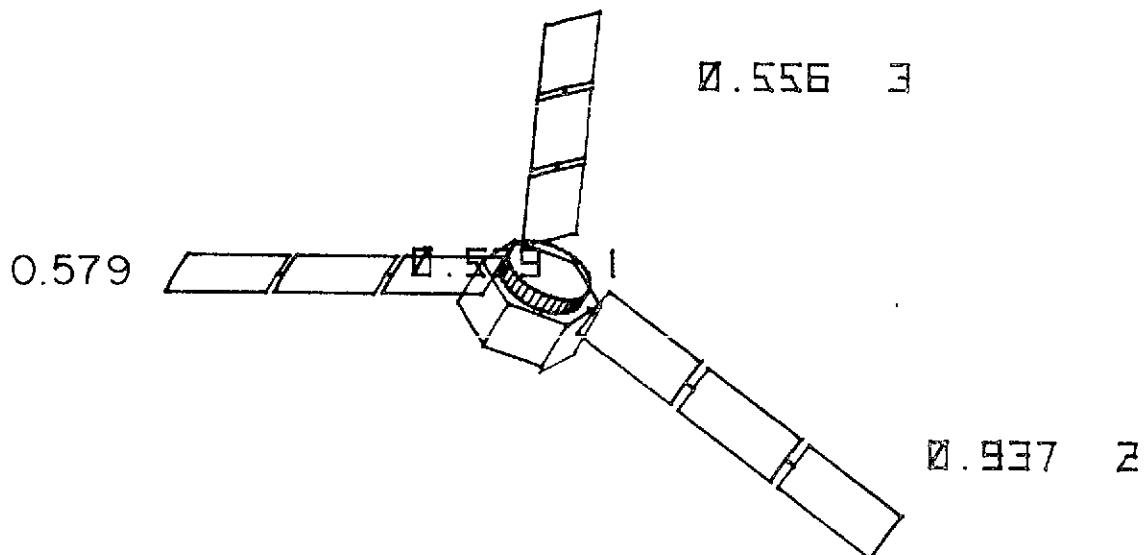
P(2) = 70

PS1 = -40

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS $2.07 - (1/12) (.502) = 2.03$



TOTAL = 2.07

P(1) = 90

PH1 = 35

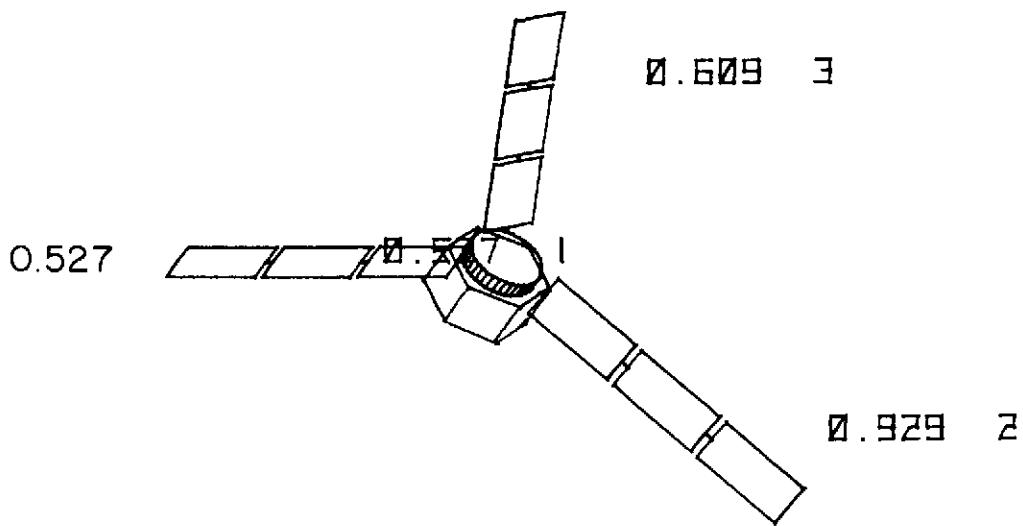
P(2) = 70

PS1 = -45

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS $2.07 - (1/12) (.556) = 2.02$



TOTAL = 2.06

P(1) = 90

PH1 = 35

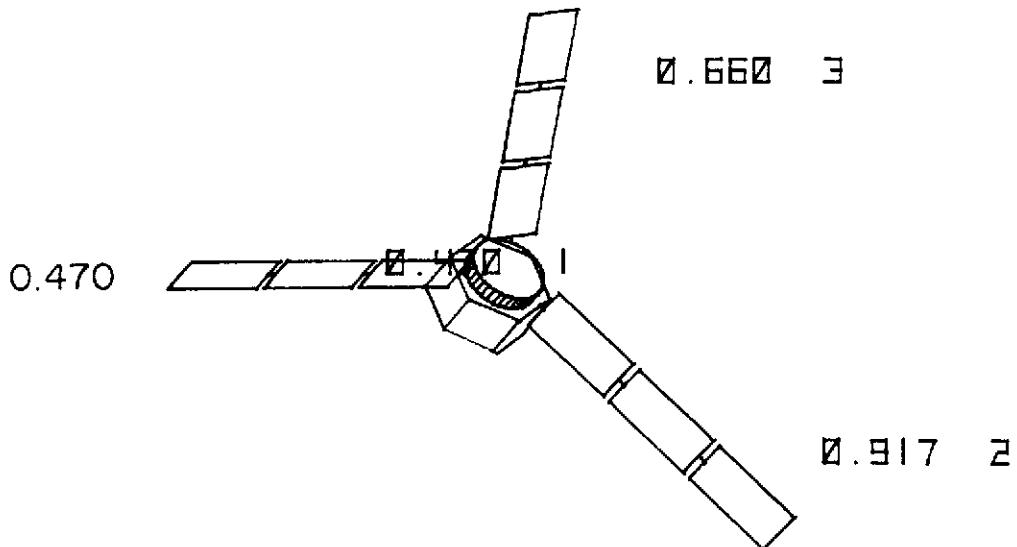
P(2) = 70

PS1 = -50

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 2.06 - (1/12) (.609) = 2.01



TOTAL = 2.05

P(1) = 90

PH1 = 35

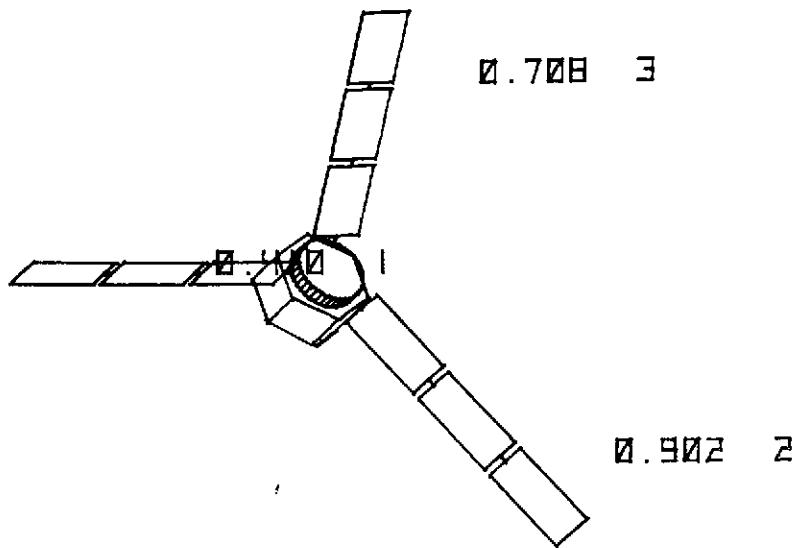
P(2) = 70

PS1 = -55

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 2.05 - (1/12) (.660) = 2.0



TOTAL = 2.02

P(1) = 90

PH1 = 35

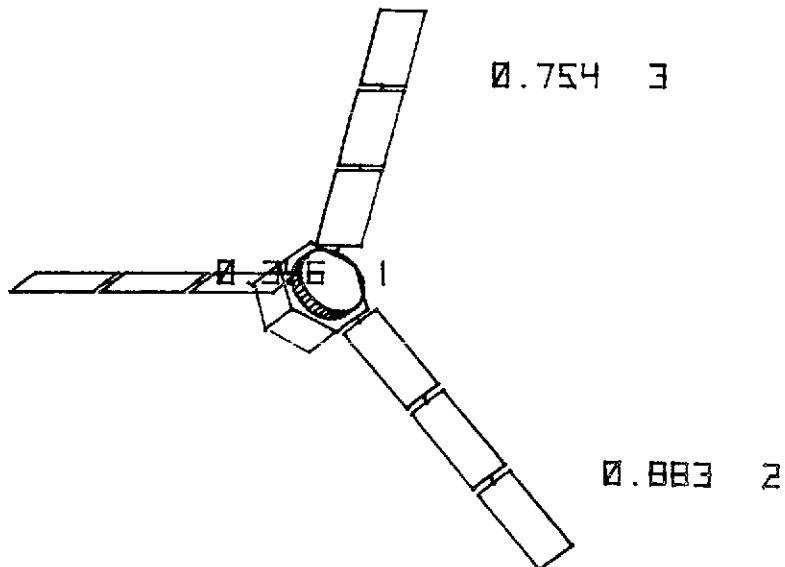
P(2) = 70

PS1 = -60

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 2.02 - (1/12) (.708) = 1.96



TOTAL = 1.98

P(1) = 90

PH1 = 35

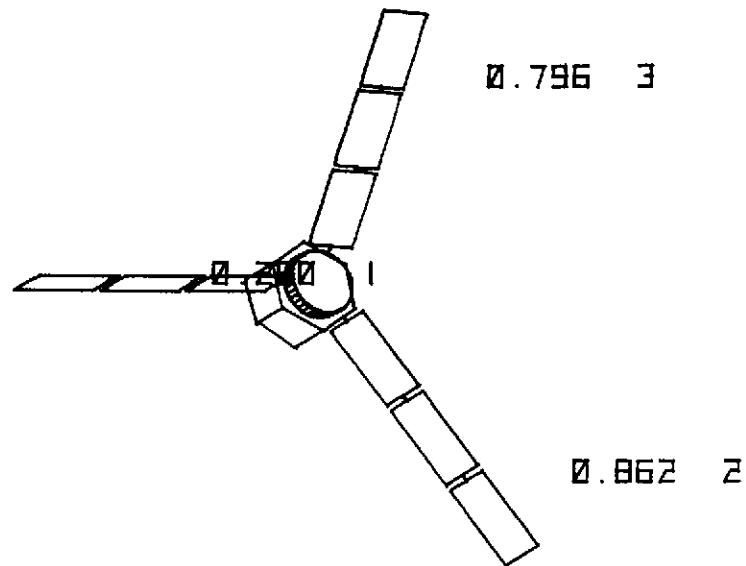
P(2) = 70

PS1 = -65

P(3) = 135

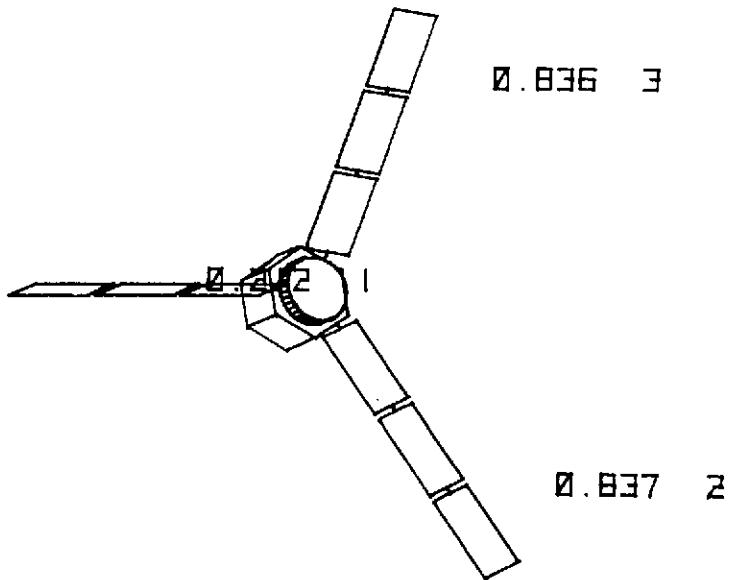
R = -90

TOTAL AFTER SHADOWING IS 1.98 - (1/12) (.754) = 1.92



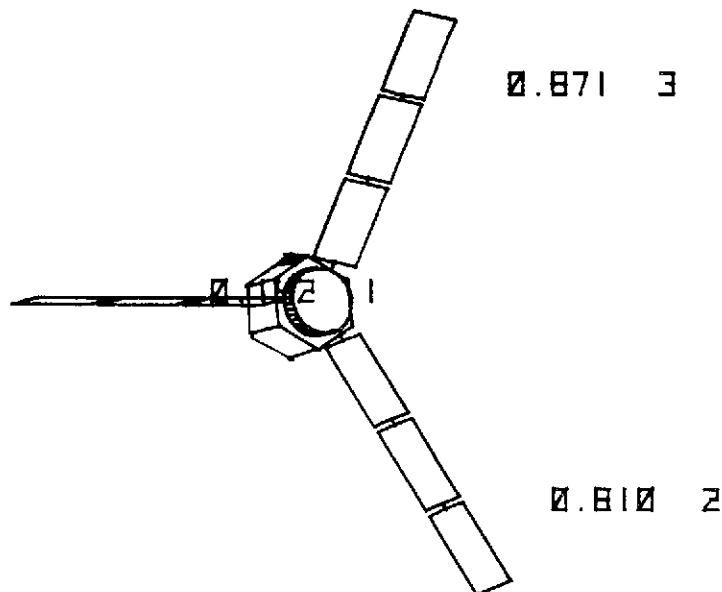
TOTAL = 1.94
 P(1)= 90 P(2)= 70 P(3)= 135
 PH1= 35 PS1=-70 R=-90

TOTAL AFTER SHADOWING IS $1.94 - (1/12) (.796) = 1.87$



TOTAL = 1.88
 P(1)= 90 P(2)= 70 P(3)= 135
 PH1= 35 PS1=-75 R=-90

TOTAL AFTER SHADOWING IS $1.88 - (1/12) (.836) = 1.81$



TOTAL = 1.82

P(1) = 90

PH1 = 35

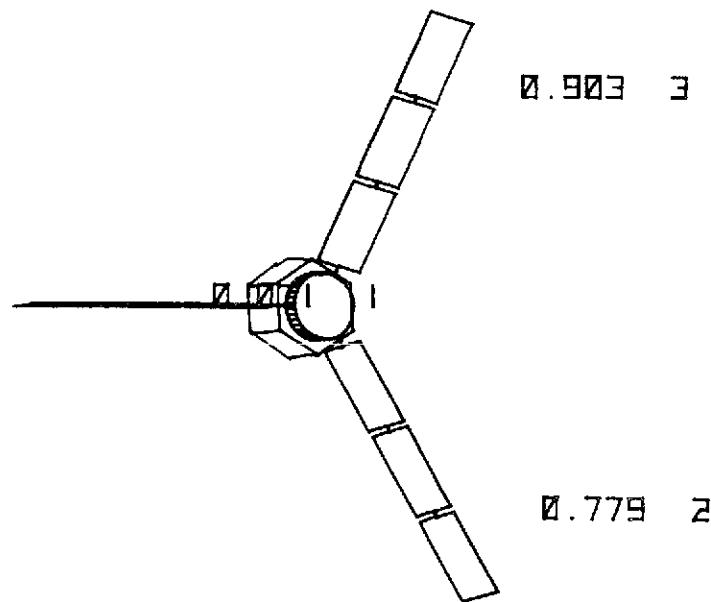
P(2) = 70

PS1 = -80

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS $1.82 - (1/12) (.871) = 1.75$



TOTAL = 1.75

P(1) = 90

PH1 = 35

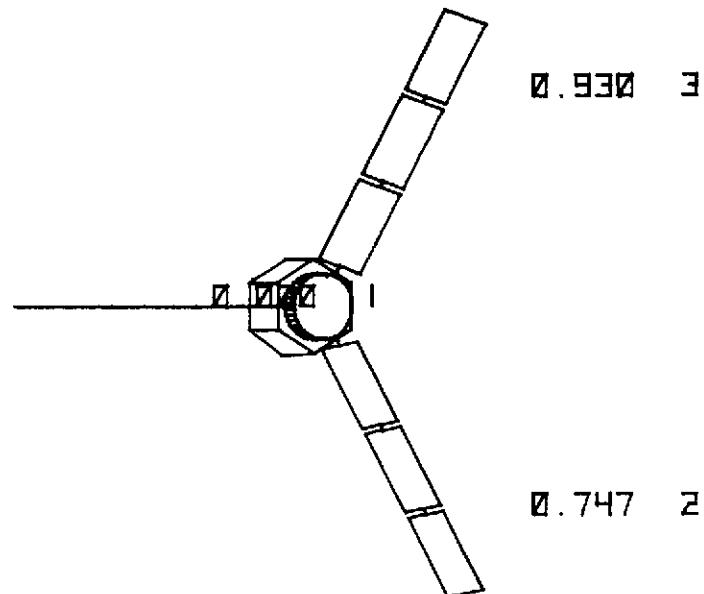
P(2) = 70

PS1 = -85

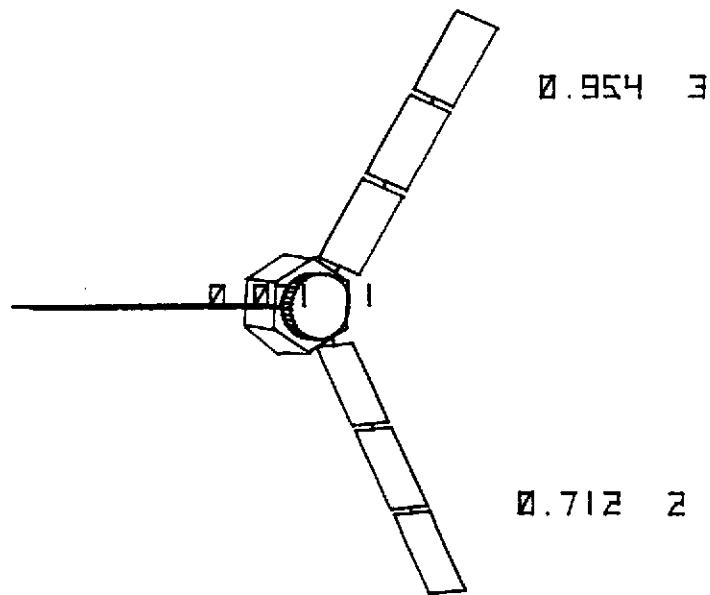
P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS $1.75 - (1/12) (.903) = 1.67$

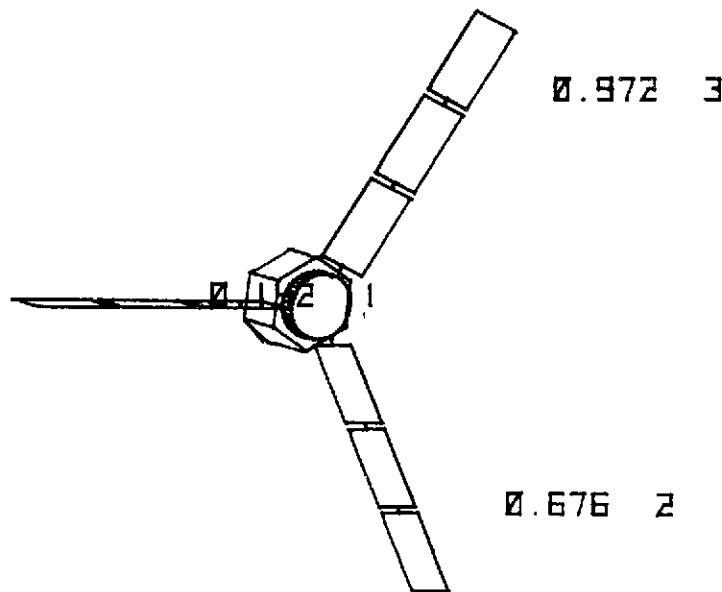


TOTAL = 1.68
P(1)= 90 P(2)= 70 P(3)= 135
PH1= 35 PS1=-90 R=-90



TOTAL = 1.74
P(1)= 90 P(2)= 70 P(3)= 135
PH1= 35 PS1=-95 R=-90

TOTAL AFTER SHADOWING IS 1.74 - (1/12) (.712) = 1.68



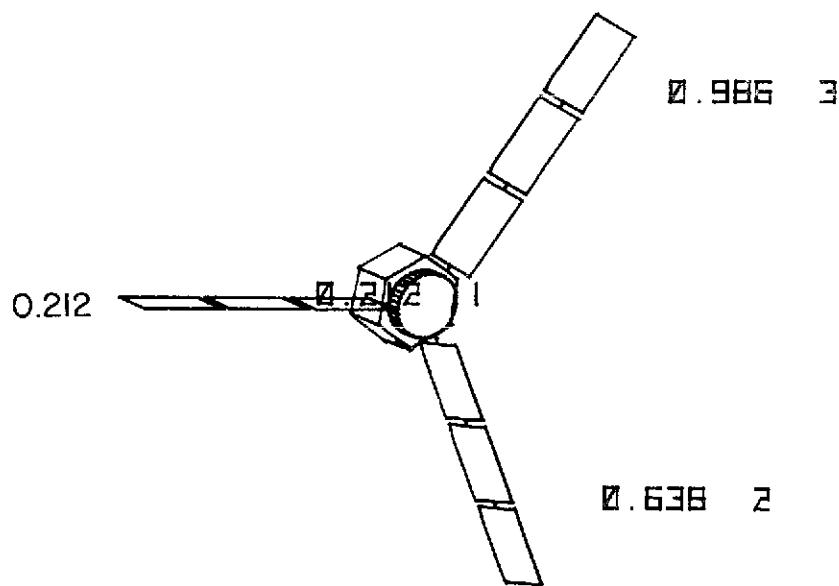
TOTAL = 1.79

P(1) = 90
PH1 = 35

P(2) = 70
PS1 = -100

P(3) = 135
R = -90

TOTAL AFTER SHADOWING IS $1.79 - (1/12) (.676) = 1.73$



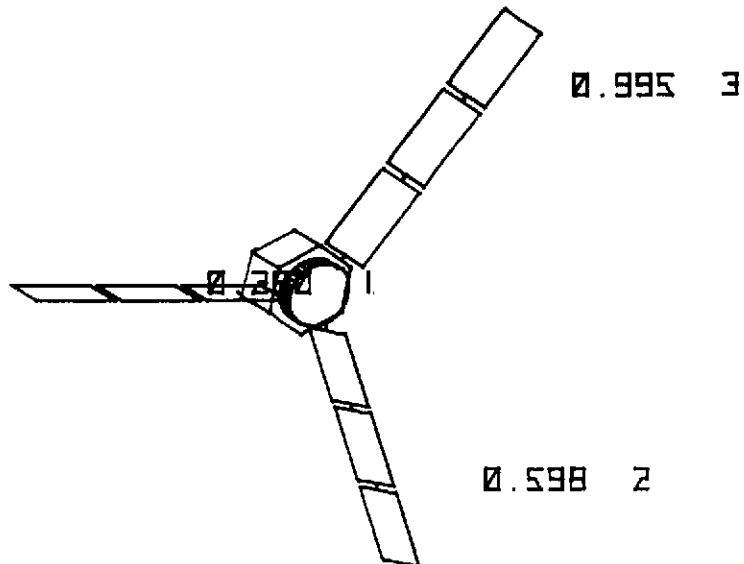
TOTAL = 1.84

P(1) = 90
PH1 = 35

P(2) = 70
PS1 = -105

P(3) = 135
R = -90

TOTAL AFTER SHADOWING IS $1.84 - (1/12) (.638) = 1.79$



TOTAL = 1.87

P(1) = 90

PH1 = 35

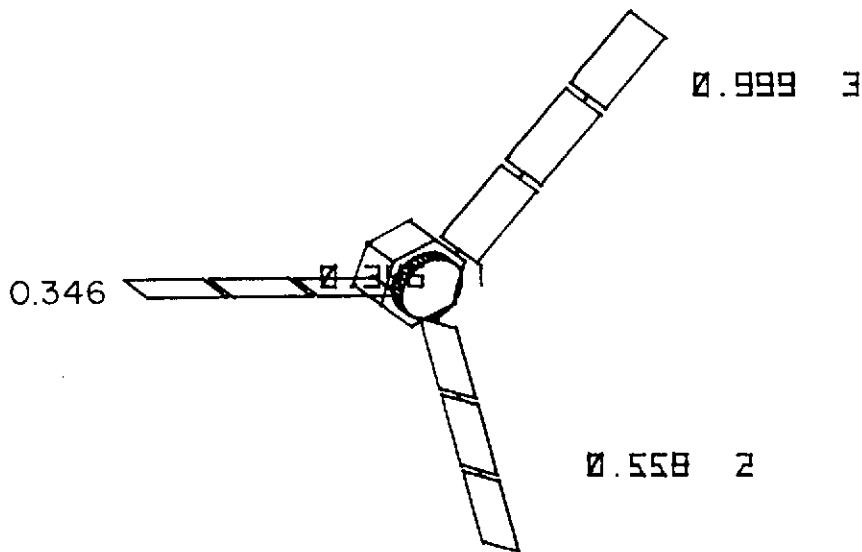
P(2) = 70

PS1 = -110

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS $1.87 - (1/12) (.598) = 1.82$



TOTAL = 1.90

P(1) = 90

PH1 = 35

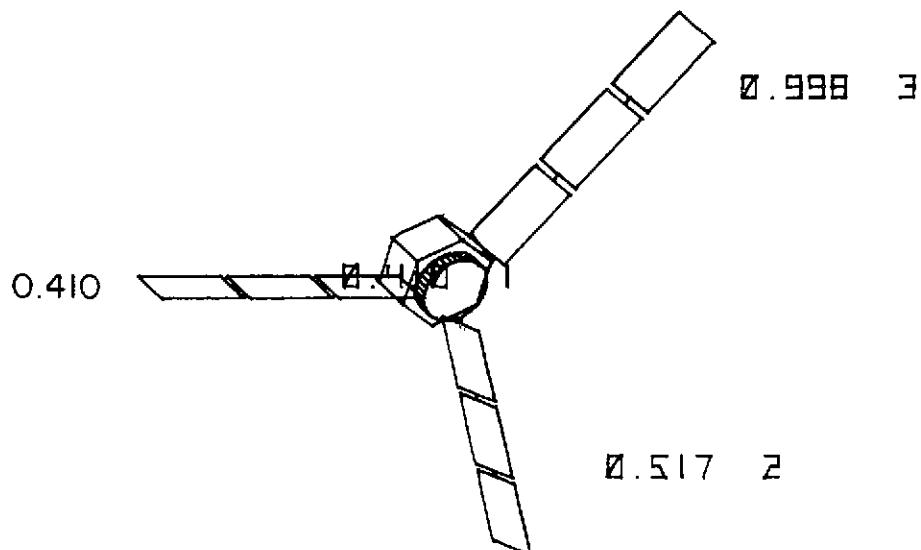
P(2) = 70

PS1 = -115

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS $1.90 - (1/12) (.558) = 1.85$



TOTAL = 1.92

PC(1)= 97

PHI = 35

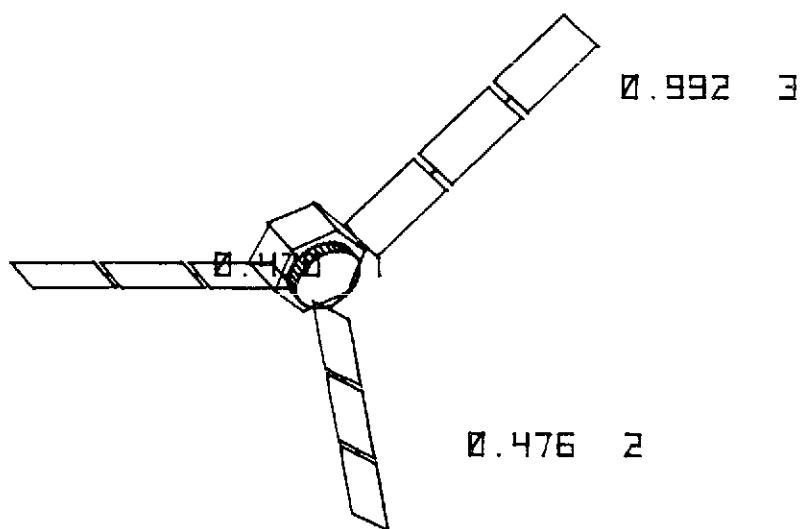
$$P(2) = 74$$

PSI = -120

$$P(3) = 135$$

$$R = -90$$

TOTAL AFTER SHADOWING IS $1.92 - (1/12) (.517) = 1.88$



TOTAL = 1.94

P(1)= 90

PHI = 35

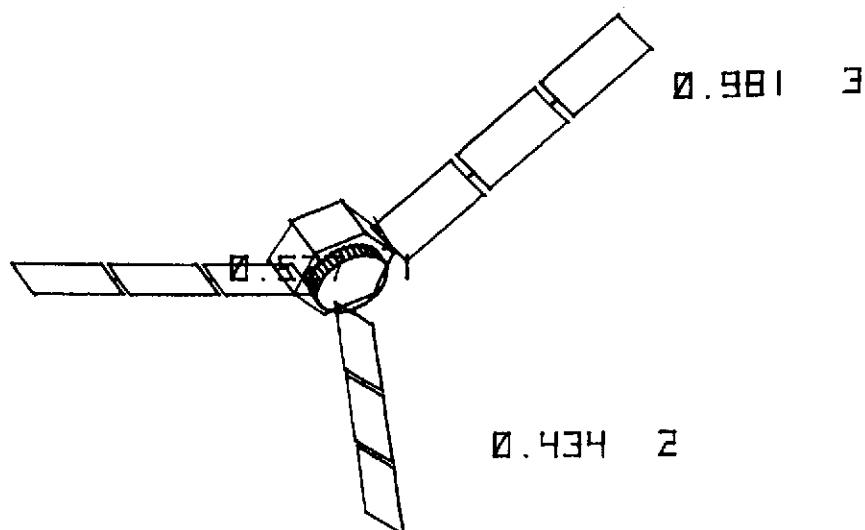
$$P(2) = 7\pi$$

PSI = 125

$P(E) = 1.35$

$$R = -9\sqrt{3}$$

TOTAL AFTER SHADOWING IS $1.94 - (1/12) (.476) = 1.90$



TOTAL = 1.94

P(1) = 90

PHI = 35

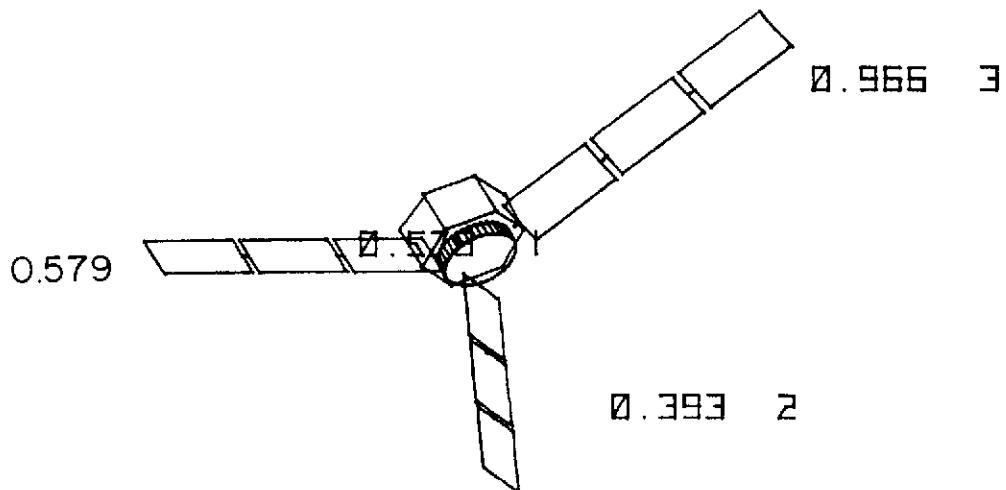
P(2) = 70

PSI = -130

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.94 - (1/12) (.434) = 1.90



TOTAL = 1.94

P(1) = 90

PHI = 35

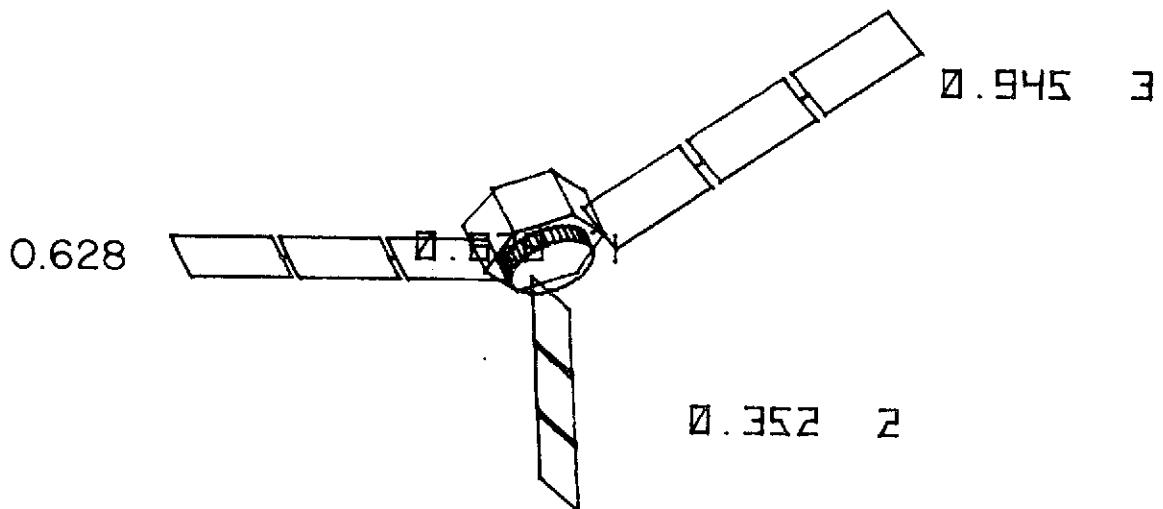
P(2) = 70

PSI = -135

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.94 - (1/12) (.393) = 1.91



TOTAL = 1.92

P(1)= 90

PHI= 35

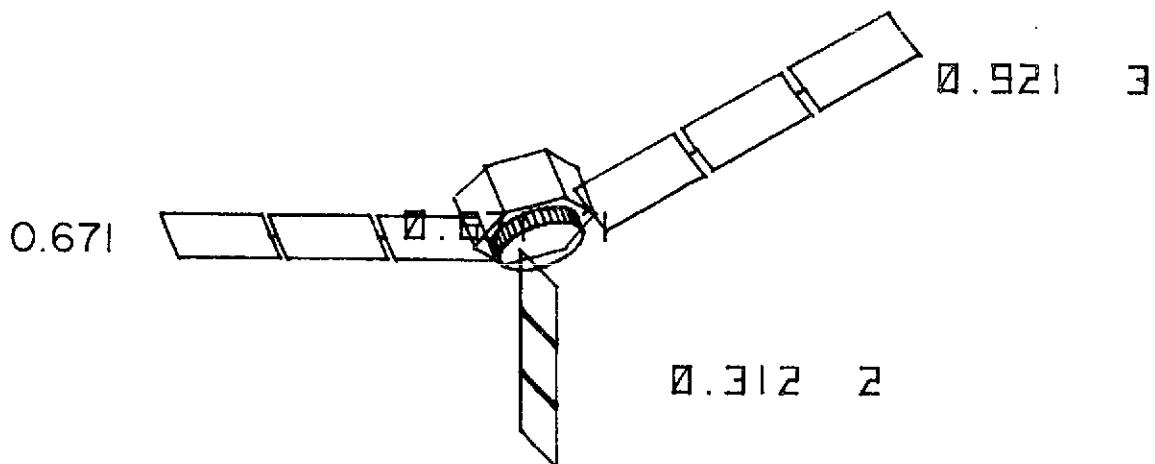
P(2)= 70

PSI=-140

P(3)= 135

R=-90

TOTAL AFTER SHADOWING IS 1.92 - (1/12) (.352) = 1.89



TOTAL = 1.90

P(1)= 90

PHI= 35

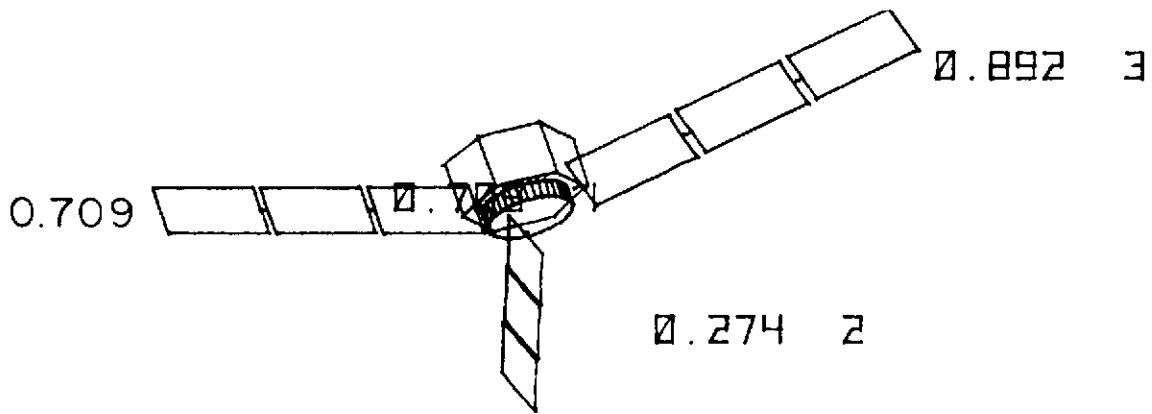
P(2)= 70

PSI=-145

P(3)= 135

R=-90

TOTAL AFTER SHADOWING IS 1.90 - (1/12) (.312) = 1.87



TOTAL = 1.87

P(1) = 90

PHI = 35

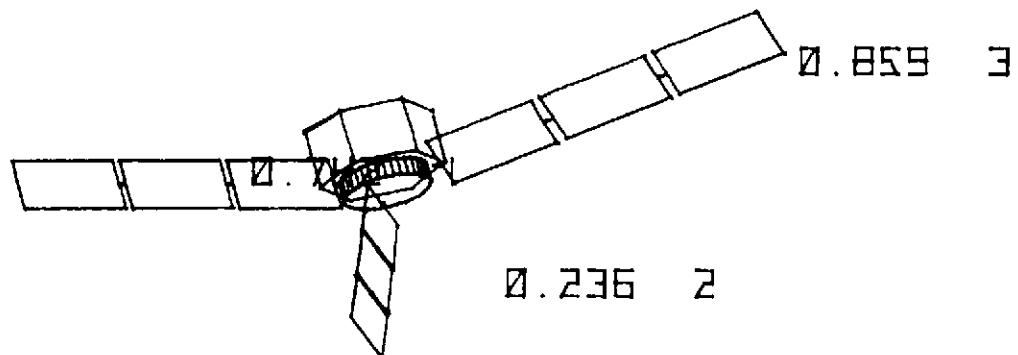
P(2) = 70

PSI = -150

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.87 - (1/12) (.274) = 1.85



TOTAL = 1.84

P(1) = 90

PHI = 35

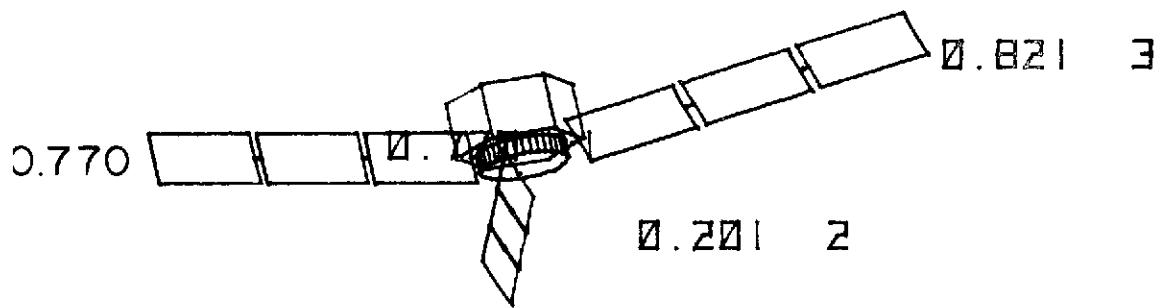
P(2) = 70

PSI = -155

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.84 - (2/12) (.236) = 1.80



TOTAL = 1.79

P(1) = 90

PH1 = 35

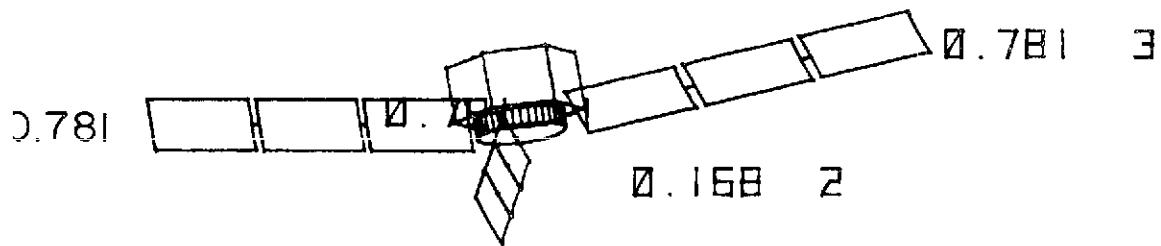
P(2) = 70

PS1 = -160

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.79 - (3/12) (.201) = 1.74



TOTAL = 1.74

P(1) = 90

PH1 = 35

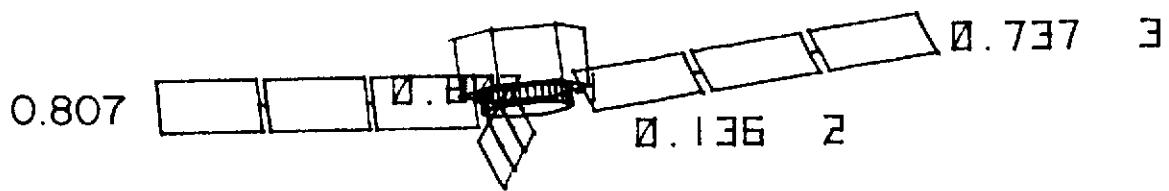
P(2) = 70

PS1 = -165

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.74 - (4/12) (.168) = 1.68



$$\text{TOTAL} = 1.68$$

$$P(1) = 90$$

$$\text{PHI} = 35$$

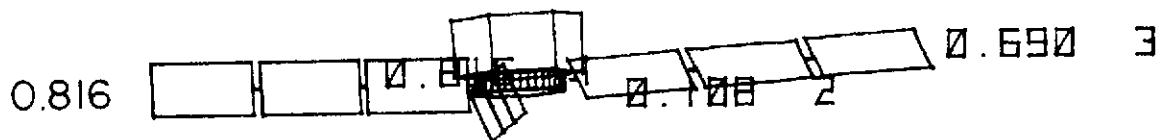
$$P(2) = 70$$

$$PSI = -170$$

$$P(3) = 135$$

$$R = -90$$

TOTAL AFTER SHADOWING IS $1.68 - (6/12) (.136) = 1.61$



$$\text{TOTAL} = 1.61$$

$$P(1) = 90$$

$$\text{PHI} = 35$$

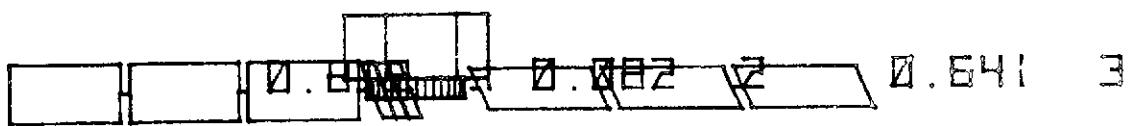
$$P(2) = 70$$

$$PSI = -175$$

$$P(3) = 135$$

$$R = -90$$

TOTAL AFTER SHADOWING IS $1.61 - 0.108 = 1.50$



TOTAL = 1.54

P(1) = 90

PH1 = 35

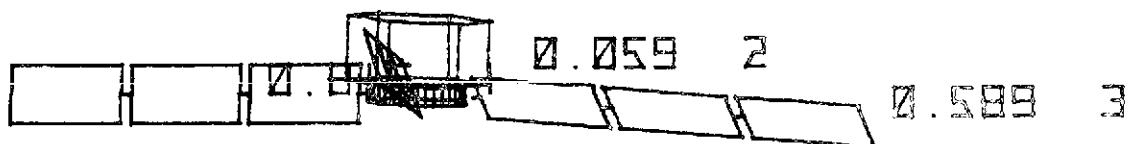
P(2) = 70

PS1 = -180

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.54 - 0.082 = 1.46



TOTAL = 1.46

P(1) = 90

PH1 = 35

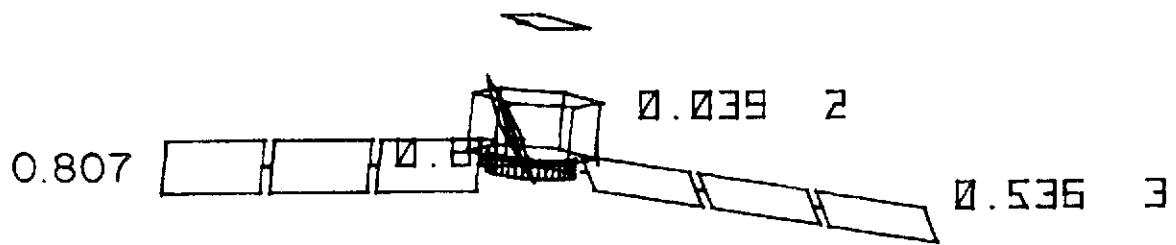
P(2) = 70

PS1 = -185

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.46 - 0.059 = 1.40



TOTAL = 1.38

P(1) = 90

PH1 = 35

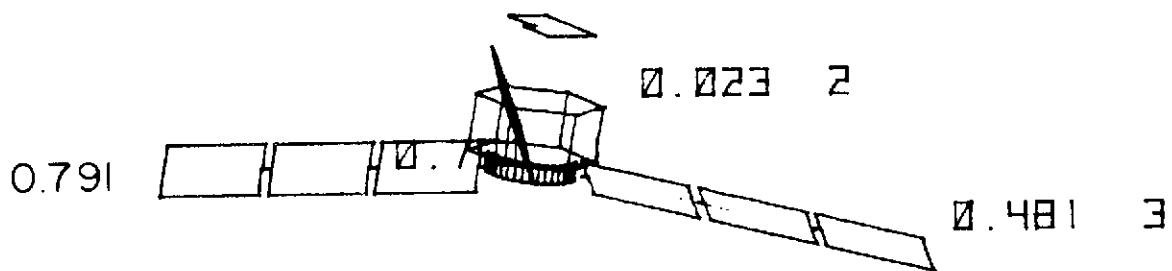
P(2) = 70

PS1 = -190

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.38 - 0.039 = 1.34



TOTAL = 1.29

P(1) = 90

PH1 = 35

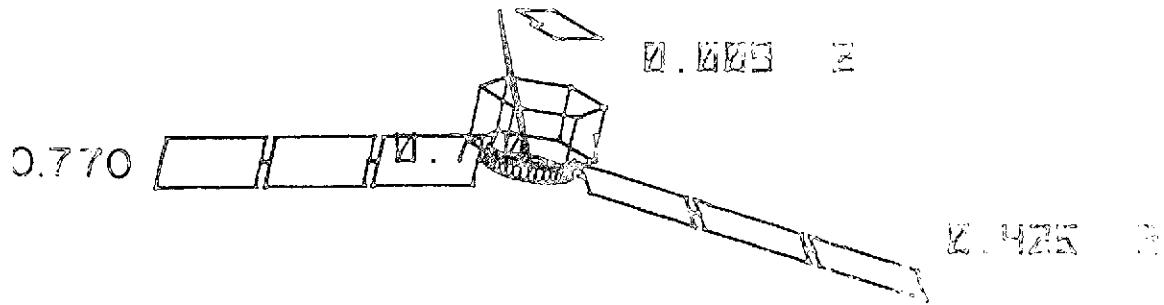
P(2) = 70

PS1 = -195

P(3) = 135

R = -90

TOTAL AFTER SHADOWING IS 1.29 - (9/12) (.023) = 1.24



$$\text{TOTAL} = 1.20$$

$$P(1) = 90$$

$$PH1 = 35$$

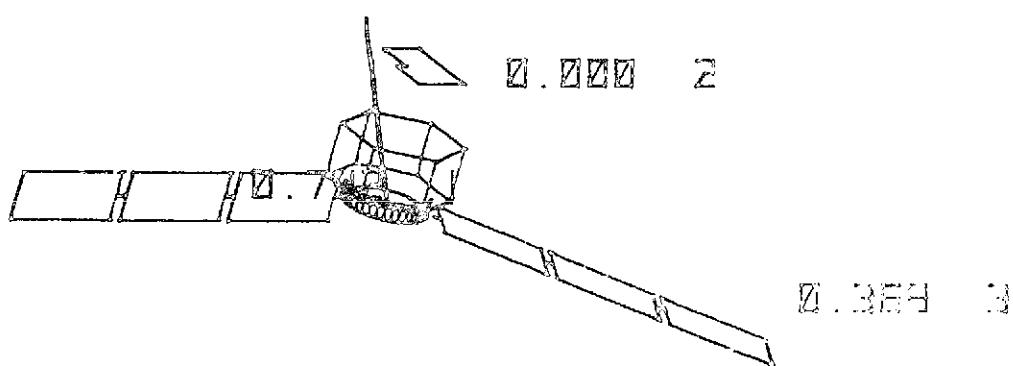
$$P(2) = 70$$

$$PS1 = -200$$

$$P(3) = 125$$

$$R = -300$$

TOTAL AFTER SHADOWING IS $1.20 - (8/12) (.009) = 1.19$



$$\text{TOTAL} = 1.11$$

$$P(1) = 90$$

$$PH1 = 35$$

$$P(2) = 70$$

$$PS1 = -205$$

$$P(3) = 125$$

$$R = -310$$